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Contributions to Education

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THE TEACHING OF GENERAL MATHEMATICS IN THE SECONDARY SCHOOLS OF THE UNITED STATES

A STUDY OF THE DEVELOPMENT AND PRESENT STATUS OF GENERAL MATHEMATICS

BY

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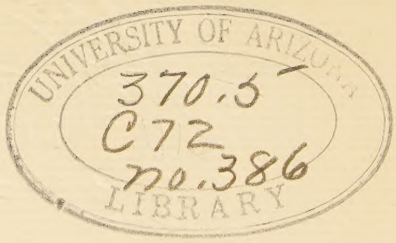

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THE TEACHING OF GENERAL MATHEMATICS IN THE SECONDARY SCHOOLS OF THE UNITED STATES

CHAPTER I

INTRODUCTION

REASONS FOR GENERAL MATHEMATICS

With the popularizing of public education has come an increased interest in the content, organization, and methods of teaching mathematics. The new psychology places emphasis upon that education which will meet the needs of the people of to-day without reference to the subject matter which has been traditionally taught. This has led to a complete reorganization of many of our courses of study. At the beginning of the present century, almost every teacher of mathematics attempted to bring about a logical development of the subject in the minds of the pupils. A few of the leading specialists of that time saw that pupils did not always learn best in this way, and they began to try to reorganize the subject upon a psychological basis. This was the beginning of the movement for general mathematics. Such a type of reorganization has been attempted in many places during the last quarter century. As a result, many changes have been made in our mathematics courses.

General mathematics has assumed different names, according to the particular type advocated. There has been so much misunderstanding of the terms used that it is necessary to reach an agreement as to what meanings will be attached to them in the present study. The different ways of uniting the subject matter and the words commonly used to designate each of them are discussed in Chapter II. The present investigation is concerned primarily with the mathematics of the ninth and tenth grades. The study must extend down into the seventh and eighth grades, however, because of the intimate relation between the mathematics of these two years and that of the ninth grade, especially in the junior high school.

HISTORY OF GENERAL MATHEMATICS

It is difficult to understand the present agitation for a general mathematics course without knowing something about the introduction and history of this type of mathematics. This is discussed in Chapter III. Some agitation for the unity of the subject matter began before 1900, but no outstanding attempt was made during the nineteenth century to unite the different branches. At the conclusion of Chapter III, some of the leading causes of the rise and success of general mathematics are set forth.

SUCCESS OF GENERAL MATHEMATICS

Ever since any type of general mathematics has been offered in this country, educators have been asking the question, "Is it a success as a subject for high school and elementary school pupils?" No absolute proof has yet been made, but certain experiments have been carried on for the purpose of comparing its success with that of the traditional type of mathematics. Some of these are rather significant. A summary of them is given in Chapter IV. The writer conducted a study of the marks in mathematics made by freshmen in the University of Minnesota who had studied general mathematics in high school. It was hoped that the marks made by these students might reveal some indication of the relative merits of the two types of mathematics. The writer also made a comparison of students in this university on the basis of the relative numbers who had general or traditional mathematics in the ninth and tenth grades and elected mathematics in the eleventh and twelfth grades and in the first year of college. A similar investigation was made of students in a large number of high schools who elected mathematics in the eleventh and twelfth grades. These investigations are reported in Chapter IV.

AN INVESTIGATION OF TEXTBOOKS

No reform of the subject matter will extend far if it does not effect a change in the textbooks and the courses of study in that particular field. To determine the effect of general mathematics upon textbooks, the writer has made a study of one hundred six textbooks and of a number of the leading courses of study in mathematics for the seventh to tenth grades inclusive. A report of the study of textbooks is given in Chapters V, VI, and VII, and of courses of study in Chapter VIII.

OPINIONS OF TEACHERS AND EXPERTS

Not all of the changes brought about in the choice and arrangement of the subject matter are due to the authors of textbooks and the writers of courses of study. In fact, these people reflect to a large extent the wishes of the teachers in the field and the best thought of the specialists in the teaching of the subject. Hence the investigation should include a study of the opinions of these two different types of educators. The writer sent out questionnaires to teachers of high school mathematics in order to ascertain their opinions in regard to the relative merits of traditional and general mathematics. A careful study was made of the replies, and a somewhat condensed report of them is given in Chapter IX. The opinions of experts have been secured from periodicals and books and in certain cases by personal requests. Chapter X gives a report of this part of the study.

CONCLUSIONS AND SUGGESTIONS FOR TEACHERS

An investigation of general mathematics would not be complete without certain conclusions and suggestions for teachers. Throughout this book, specific conclusions are given from time to time. In the last chapter (Chapter XI), a summary of the most important findings is given, as well as a list of definite suggestions for teachers which the writer thinks is warranted as a result of the careful study he has made of the situation.

CHAPTER II

UNITING THE SUBJECT MATTER OF MATHEMATICS

THE TRADITIONAL ARRANGEMENT

The traditional arrangement of mathematics in this country is arithmetic with the geometry of mensuration in the elementary grades, algebra in the ninth grade, and plane geometry or another year of algebra in the tenth grade. Beyond the tenth grade, there has been no such uniformity. Various subjects such as one year of plane geometry where it is not taught in the tenth grade, one-half year or one year of intermediate algebra, one-half year of solid geometry, one-half year of trigonometry, one-half year of arithmetic, and one-half year of college algebra are among the subjects which have found favor in many places. In recent years, statistics, analytic geometry, and calculus are being introduced in a few high schools.

OTHER ARRANGEMENTS

For the last thirty-five years, certain educators have maintained that arithmetic, algebra, and geometry should not be taught in separate "water-tight compartments," but should be interrelated in some way. This has led to such terms as unified, correlated, general, parallel, fused, composite, mixed, combined, and co-operative mathematics. Some of these terms have been introduced to indicate a new way of treating mathematics, but nearly all of them have been used by mathematicians in such a way that there have been no clear distinctions in the meanings of the different terms used.

PARALLEL MATHEMATICS

The only type of mathematics, except the traditional arrangement, which has been rather clearly understood by the great majority of teachers, is the parallel type. The most common illustration is the teaching of algebra and geometry side by side, algebra being offered two or three days each week and geometry

on the other days. This gives the teacher a chance to develop the easier portions of each subject before taking up the more difficult parts. When these subjects have been treated in this way, it has usually been done in the first two years or in the second and third years of the four-year high school. Even though the subjects are taught separately, there is a greater tendency to emphasize the relationships of the two subjects than in the traditional arrangement. This method of teaching algebra and geometry undoubtedly has contributed to the fusing or uniting of the subjects.

CORRELATED MATHEMATICS

Perhaps the best way to describe correlated mathematics is to quote from the preface of *First-Year Mathematics* by E. R. Breslich:

> The relationship between algebra and geometry has long been recognized by technical students of mathematics, quite apart from any consideration of the desirability of teaching them together. Algebra and geometry supplement each other. Both are used to express facts about quantity; *e.g.*, the graph and the formula both express the law of a group of numerical facts. Both give these facts in a form easily taken in by the trained eye; both state the facts in generalized form and thus make the deduction of any number of particular instances possible. By correlating the two related forms of thought in a single course of instruction the student's comprehension of quality is at the same time simplified and deepened; simplified, because the double method of attack makes it easier to overcome difficulties; deepened, because of the more enduring impression made upon the mind. In the course presented in this book, geometry is used throughout the book to illustrate algebraic processes, while algebra carries on the reasoning in the compact and abstract symbols which generalize quantitative facts in a degree which is impossible in graphic expression.
>
> . . . . . .
>
> When the various branches of mathematics are treated as separate subjects, there is a tendency for each to take on the rigid form of the final science. This tends inevitably to a certain formalism in mode of presentation. Such formalism is not the best method for the beginner. Correlation helps to avoid excessive formalism.[1]

The very emphasis placed by the advocates of correlated mathematics upon correlation has led to a very closely united or fused course in mathematics in the high school. Whether this has produced a too close and somewhat forced union of the subjects in mathematics is a debatable question. It has apparently more

[1] E. R. Breslich, *First-Year Mathematics*. University of Chicago Press (1916).

nearly broken down the distinction between the different subjects than any other type developed in our public schools.

GENERAL MATHEMATICS

General mathematics first originated as a somewhat less highly correlated course in mathematics. It represented an effort to select the best from the different subjects, present it in such a way that the pupil of average intelligence can learn it, and place emphasis upon correlation only to the extent that the correlation represents the natural relationships of the subjects. In the preface of his book on general mathematics, Professor W. D. Reeve makes the following statement:

> The aim of the general mathematics course is to give the student *not so much mathematics* as is generally given in the traditional courses but *more about mathematics*. In other words, the student should know mathematics as a tool to be used rather than as a group of related subjects, such as algebra, geometry, trigonometry, and so on.[2]

In the preface of *General High School Mathematics* by Smith, Foberg, and Reeve, the authors have stated their own views of general mathematics as follows:

> For the work of the first year this view of mathematics should not be limited, as in the traditional plan, to algebra or any other single branch. It should give an appreciation of the nature and uses of algebra, and acquaintance with the facts of geometry that are included in what has come to be called "intuitive geometry," and an insight into the simple practical uses of trigonometry. . . .
>
> The third principle is that a student is most successful if he studies one thing at a time, and for a sufficient length of time to acquire at least some feeling of mastery. It is here that many textbooks fail. The practice of selecting from the broad field of mathematics a little here and a little there, without any attention to a proper sequence, is based upon an erroneous idea of the meaning of correlation, and has been responsible for the short life of many books in this field. As the successful teacher knows, a proper view of the nature of correlation seeks points of genuine contact between any two parts of the science. The fact that two branches of study are both classified under the general name of mathematics is no reason for assuming that there is any close relation between them. The authors have not sought, merely for the sake of novelty, to mix the various topics of a science in which compartments naturally exist. They have, as will be seen, perfect sympathy with scientific correlation, and have adopted it, but they have no sympathy whatever with an unnatural and unscientific mixing or fusing of unrelated topics.[3]

[2] W. D. Reeve, *General Mathematics*, Book II. Ginn and Company (1922).

[3] D. E. Smith, J. A. Foberg, and W. D. Reeve, *General High School Mathematics*, Book I. Ginn and Company (1925).

Another exponent of general mathematics has written:

> General mathematics is an introductory, basic, exploratory course in which the simple and significant principles of arithmetic, algebra, intuitive geometry, statistics, and numerical trigonometry are taught so as to emphasize their natural and numerous interrelations.[4]

General mathematics is concerned with the psychological rather than the logical arrangement of subject matter. It attempts to give the pupils the best mathematics available regardless of which branch it happens to be. Correlated mathematics, already described, is really a particular kind of general mathematics.

COÖPERATIVE MATHEMATICS

A modification of general mathematics has recently been suggested by Fletcher Durell. He describes a kind of mathematics which he calls coöperative mathematics:

> Many teachers of mathematics, either from personal preference, or because of curriculum and similar difficulties beyond their control, have been unable to go the whole length with the so-called general or fused mathematics. Yet almost all mathematical teachers who have seriously and earnestly studied and experimented with general mathematics, have learned many details from the method which they have been able profitably to use as modifications of the traditional method of teaching the different branches of mathematics separately.
>
> The general result with such teachers has been a tendency toward a form of presentation which it is here proposed to name Coöperative Mathematics.
>
> In coöperative mathematics, each branch as arithmetic, algebra, geometry, etc., keeps its own individuality and organization, but at the same time it transfers from the other branches and welcomes to itself any detail which may be logically useful. It freely and aggressively uses such transferred material, provided always that the transferred part does not impair, but rather, if possible, aids the individual organization of the part to which it has been transferred.[5]

CONCLUSION

From these descriptions, it appears that correlated mathematics is the most closely united or fused of the different kinds of mathematics already described, that the so-called general mathematics is next in order, and that coöperative mathematics, as described by Durell, is somewhat less united than general mathematics. Unified mathematics is a term which has been used to indicate a course built around some unifying principle such as

[4] R. Schorling, "General Mathematics," *The Mathematics Teacher*, 20:65-81 (1927).

[5] Fletcher Durell, "Coöperative Mathematics," *School Science and Mathematics*, 27:905-9 (1927).

the function concept; combined or composite mathematics may mean any general course in mathematics; and fused mathematics has generally meant the same as correlated mathematics. To the average teacher, the different names are confusing and sometimes misleading, and it is apparent that no real purpose is served by a sharp distinction between them. A careful study of textbooks and courses of study shows such a unity of purpose and similarity in procedure that it does not seem best in this study to make any fine distinction between the different types. Any type of mathematics which attempts to break down the compartment method of presentation of the subject matter and to teach mathematics from a psychological point of view will be classed in this study as general mathematics.

CHAPTER III

HISTORY OF THE GENERAL MATHEMATICS MOVEMENT

In the preceding chapter, the different ways of uniting the various subjects of mathematics have been discussed. It is now proposed to show how the present traditional arrangement of mathematics came to be introduced into our schools, to trace the movements for correlated and general mathematics in particular, and to relate briefly the causes of its rise and success.

THE ELEMENTS OF EUCLID

It has been very commonly believed that *Euclid's Elements*, written about 300 B.C., is merely the earliest edition of our present course in demonstrative geometry. Although a part of the *Elements* has determined largely the content and nature of the present course, it should not be thought that the *Elements* is merely an early form of our high school geometry. The *Elements* is the elements of all mathematical knowledge of Euclid's time, and presents the achievements of different mathematicians who followed various paths of investigation. The Greeks were particularly interested in geometry, however, and all of their mathematical studies were made from a geometric point of view. The *Elements* consists of thirteen books which treat the following:

I. Congruence, parallels, the Pythagorean Theorem;
II. Identities which we would now treat algebraically, like $(a + b)^2 = a^2 + 2ab + b^2$, but which were then treated geometrically; areas; the Golden Section;
III. Circles;
IV. Inscribed and circumscribed polygons;
V. Proportion treated geometrically; in part, a geometric way of solving fractional algebraic equations;
VI. Similarity of polygons;
VII–IX. Arithmetic (the ancient theory of numbers) treated geometrically;
X. Incommensurable magnitudes;
XI–XIII. Solid geometry.[1]

[1] D. E. Smith, *History of Mathematics*, Part I, p. 106. Ginn and Company (1923).

An examination of the contents reveals that Books I, III, IV, and VI, and part of Book II constitute mainly our present course in plane geometry; Books XI–XIII our present solid geometry; the subject matter of Book V and part of Book II is material which is now treated algebraically; Books VII–IX are pure arithmetic when taken out of the geometric setting as has now been done; and Book X treats of the arithmetical characteristics of divisions of straight lines (i.e., irrationals). There is a growing tendency, as an investigation of current textbooks will show, to go back to the methods of Euclid for the purpose of supplementing and illustrating our present algebraic treatment of such identities as those contained in Book II. For Euclid, the unifying thread in all of the diverging ideas of mathematics appears to have been number. It was, however, almost if not entirely impossible for him to think of number without thinking of it as the length of a line, as an area, or as some other geometric magnitude.

MATHEMATICS IN THE EARLY AMERICAN COLLEGES

It is not the purpose of this study to trace the development of mathematics from the time of Euclid to the present day. The *Elements* is mentioned to show that there was no attempted separation of mathematics into compartments at the time of the early development of demonstrative geometry. Likewise, the early history of arithmetic and algebra which were developed chiefly by the Hindus and Arabians shows that there was no attempt to separate the subjects.

It would be easier to understand the recent movements for the unifying of mathematics if we knew how we happened to have our compartment plan of teaching mathematics. Since American high school mathematics had its beginning in the American colleges, it is necessary to know something about the early history of mathematics in these colleges. The first college teachers were trained in England and Scotland and so naturally introduced the subject matter and plan of the English universities. Therefore, in the study of the American colleges, the plan which was followed in England will be revealed.

In 1643, no mathematics was required for entrance to Harvard University, and the only mathematics offered was a year of arithmetic and geometry in the senior (third) year.[2] Algebra was

[2] H. O. Rugg and J. R. Clark, *Scientific Method in the Reconstruction of Ninth-Grade Mathematics*, pp. 38-41. University of Chicago Press (1918).

probably not introduced until the beginning of the eighteenth century. Two early algebra notebooks written by Harvard students are known to be in existence to-day.[3] One was written about 1730 and the other, very much like the first, about 1738. Ward's *Mathematics* was used as a textbook at this time.[4] This book contains 142 pages devoted chiefly to arithmetic, 140 pages to algebra, 78 pages to geometry, 36 pages to conic sections, 36 pages to arithmetic of infinites, and 24 pages to practical gauging. Although the author divided the book into six parts, with the number of pages indicated above, there is considerable correlation of the subjects, as will be explained in Chapter VI. The author was interested in mathematics rather than the different branches of mathematics. In the Introduction to Part I, he stated:

> The Business of Mathematicks in all its Parts, both Theory and Practice, is only to search out and determine the true Quantity; either of Matter, Space, or Motion, according as Occasion requires. . . .
>
> The consideration of these, according as they may be proposed, are the Subjects of the Mathematicks, but chiefly that of Matter.
>
> Now the consideration of Matter, with respect to its Quantity, Form, and Position, which may either be Natural, Accidental, or Designed, will admit of infinite Varieties; But all the Varieties that are yet known, or indeed possible to be conceived, are wholly comprised under the due consideration of these Two, Magnitude and Number, which are the proper Subjects of Geometry, Arithmetick and Algebra. All other Parts of the Mathematicks being only the branches of these three Sciences, or rather their Application to particular Cases. . . .
>
> *Algebra* is a Science by which the most abstruse or difficult Problems either in Arithmetick or Geometry are Resolved and Demonstrated, that is, it equally interferes with them both; and therefore is promiscuously named, being sometimes called Specious Arithmetick. . . and sometimes its called Modern Geometry.

Thus it is apparent that in the early history of Harvard University no attempt was made to draw a sharp line of separation between the branches of mathematics. In 1787, we find more apparent separation, arithmetic being given in the freshman year and algebra in the sophomore year.

At Yale University, Alsted's *Geometry* was used as a textbook in 1719. In 1742, students were advised to study arithmetic and algebra in the freshman year and geometry in the sophomore year.

[3] Leo Genevra Simons, *Introduction of Algebra into American Schools in the Eighteenth Century*. U. S. Bureau of Education, Bulletin, 1924, No. 18.

[4] John Ward, *The Young Mathematician's Guide: Being a Plain and Easy Introduction to the Mathematics*. London (1734).

Ward's *Mathematics* was also used at Yale University as well as at Harvard University about this time. Evidently there was no strictly logical separation of the subject matter. In the list of studies at Yale University in 1778, *The Elements of Algebra* by Nathanial Hammond was included. This book was devoted entirely to algebra. In about one-seventh of the book, algebra and geometry were correlated. These illustrations indicate what a more thorough investigation would undoubtedly show—that, as the subjects in mathematics developed in our American colleges, there was a tendency to separate the subject matter more and more until by the beginning of the nineteenth century the subject matter of arithmetic, algebra, and geometry was fairly well defined. When the subjects became separated, arithmetic was placed first probably because of its greater usefulness and field of application, and algebra next because of its intimate relation to arithmetic.

ALGEBRA AND GEOMETRY AS ENTRANCE REQUIREMENTS

These subjects were gradually moved down into the earlier years in the college curriculum and finally became entrance requirements. In 1802, Harvard University required arithmetic to the Rule of Three for entrance; in 1820, algebra was required; and in 1844, geometry was added to the list of requirements for entrance. Yale University required arithmetic and algebra to quadratics for entrance in 1845 and the first two books of *Playfair's Euclid* in 1855. These subjects, after securing a place in the secondary schools, were moved down until they occupied their present place in the curriculum. At first, algebra and geometry were given in the third and fourth years of the preparatory schools; at the present time they are usually placed in the first and second years of the high school, and arithmetic is given in the elementary school.

When algebra and geometry became entrance subjects, the colleges designated exactly the subject matter to be covered and very often the textbooks to be used. The subject matter was that previously used in the colleges. The textbooks were generally written by college professors and often were the same ones which the colleges had used. Thus the courses of study which were developed by college professors for adult minds, and which were not at first intended for secondary school pupils, were moved down

into the preparatory schools. Under such conditions, it is quite natural that the subject matter should be arranged logically rather than psychologically. Hence our high schools have inherited courses in algebra and geometry, and even in arithmetic, which are arranged according to a systematic development of the subjects with little or no regard to the way in which pupils learn. From the point of view of one who already knows the subject, this seems to be the proper arrangement and to be the easiest to teach, especially since it is the way most persons have been taught themselves. This accounts in a large measure for the opposition to the movements to break down the strict separation of the subject matter which has resulted in "water-tight compartments."

ATTEMPTS TO MAKE THE INTRODUCTION TO ALGEBRA EASIER

Even though secondary mathematics in the nineteenth century was divided into logical compartments of arithmetic, algebra, and geometry, we should not be unmindful of the fact that certain teachers recognized the difficulty which this separation into distinct subjects caused the pupils, and that attempts were made to find ways to make the introduction to a new subject less difficult. Some authors also tried to make the transition from arithmetic to algebra easier than was the custom at that time. One early attempt to do this was made by Warren Colburn in 1825. In the preface of his algebra is the following statement:

> The first object of the author of the following treatise has been to make the transition from arithmetic to algebra as gradual as possible. The book, therefore, commences with practical questions in simple equations, such as the teacher might readily solve without the aid of algebra. This requires the explanation of only the sign plus and minus, the mode of expressing multiplication and division, the sign of equality, together with the use of a letter to express the unknown quantity. These may be understood by anyone who has a tolerable knowledge of arithmetic. All of them except the use of the letter have been explained in arithmetic.[5]

In the preface of a textbook, published in 1852, the author, Charles Davies, made the following statement:

> Although Algebra naturally follows Arithmetic in a course of scientific studies, yet the change from the methods of reasoning on numbers to a system of reasoning entirely conducted by letters and signs, is rather abrupt and not unfrequently discourages the pupil.

[5] Warren Colburn, *An Introduction to Algebra upon the Inductive Method.* Cummings, Hilliard, and Company, Boston (1825).

In this work, it has been the intention, to form a connecting link between Arithmetic and Algebra, to unite and blend, as far as possible, the reasoning on numbers with the more abstruse methods of analysis.[6]

There is much greater emphasis on the uniting of arithmetic and algebra at the present time than in the preceding century. Some recent authors, however, have not provided for the transition from one subject to another even as well as these two authors did nearly a hundred years ago.

THE REPORT OF THE COMMITTEE OF TEN

The Committee of Ten, appointed by the National Educational Association, published its report in 1893.[7] This committee recommended that a course of instruction in concrete geometry, with numerous exercises, be introduced into the grammar schools. It was suggested that in the lower grades, the instruction in this type of geometry be given informally and in the upper grades as a separate course one hour a week. The proposed course was similar to the intuitive geometry in our present seventh, eighth, and ninth grade mathematics. In it the main facts of plane and solid geometry were to be taught not as logical exercises, but in as concrete and objective a form as possible. For example, it suggested that simple properties of similar figures and similar solids should not be proved, but should be illustrated and confirmed by cutting up and rearranging drawings and models. It was further suggested that this course should include the careful construction of plane figures, both by the unaided eye and with the aid of ruler, compasses, and protractor; the indirect measurement of heights and distances by drawing to scale; and elementary mensuration. It was recommended that the pupils should learn to estimate and measure with some degree of accuracy and that the truth of the conclusions reached should be established by numerous concrete illustrations.

This same committee also recommended parallel courses in algebra and demonstrative geometry in the tenth and eleventh grades, two and one-half hours a week being given to each subject. Here again the work of the committee shows the influence of the foreign schools. The prevailing opinion in our country has been that this was the way that mathematics was offered in the

[6] Charles Davies, *Elementary Algebra*. A. S. Barnes and Company (1852).

[7] Charles W. Eliot and Others, Report of the Committee on Secondary School Studies (appointed at the meeting of the National Educational Association, July 9, 1892) with the Report of Committees. U. S. Bureau of Education (1893).

European schools at that time. This is true to a large extent, but our American educators missed for some time a strong foreign influence toward uniting the different subjects. The greatest German specialist on the teaching of mathematics at that time was Felix Klein. In the "Provisional Report of the Committee of Fifteen on Geometry Syllabus," published in 1911, was the following statement concerning Professor Klein:

> Since about 1890 the activities of Felix Klein of Göttingen in mathematical reform, have been very great. For the first time since Kästner, is the influence of university professors upon the teaching of elementary mathematics in Germany beginning to be strongly felt. Among the defects of geometrical instruction, he points out the insufficient fusion of the various branches of elementary mathematics. Thus, too little attention is given to drawing of solids and to projection, to the idea of motion in a figure to replace Euclidian rigidity, to the fusion of arithmetic and geometry, and to the introduction of the coördinate representation of analytics.[8]

Most of this strong influence in Germany which was gradually bringing about the fusion of mathematical subjects was apparently missed by the Committee of Ten. Perhaps the movement was still too new to impress sufficiently our American educators.

CONSTRUCTIONAL AND OBSERVATIONAL GEOMETRY

The idea of concrete geometry did not originate with the Committee of Ten. In the "Provisional Report of the Committee of Fifteen on Geometry Syllabus" was the following statement:

> Another movement that sprang from the teachings of Pestalozzi and Herbart was the adoption of preliminary courses on observational geometry and drawing, about 1870. Such courses had been recommended long before this time. This movement was stronger in Germany than in England or France.

Thus it appears that work in observational geometry began in Europe more than twenty years before the report of the Committee of Ten was made. Instruction in constructional geometry was going on in this country even before the report of this committee was prepared. At about the time of the report, the high school in connection with the Armour Institute in Chicago presented a course, called inventional geometry. Spencer's *Inventional Geometry* was used as a textbook.[9] A note to the publishers from Herbert Spencer, the son of the author of this book, is published in the preface, part of which is here quoted.

[8] H. E. Slaught and Others, "Provisional Report of the National Committee of Fifteen on Geometry Syllabus." *School Science and Mathematics*, 11:329-55, 434-60, 509-31 (1911).

[9] William George Spencer, *Inventional Geometry*. D. Appleton and Company (1876).

I am glad that you are about to republish, in the United States, my father's little work on "Inventional Geometry." Though it received but little notice when first issued here, recognition of its usefulness has been gradually spreading, and it has been adopted by some of the more rational science teachers in schools. Several years ago I heard of its introduction at Rugby.

To its great efficiency, both as a means of producing interest in geometry and as a mental discipline, I can give personal testimony. I have seen it create in a class of boys so much enthusiasm that they looked forward to their geometry lesson as a chief event in the week. And girls initiated in the system by my father have frequently begged of him for problems to solve during their holidays.

In the introduction to the book the author made the following brief statement as to what he considered the purpose of such a course in geometry:

Geometry may be divided into two parts—practical and theoretical: the practical bearing a similar relation to the theoretical that arithmetic does to algebra. And just as arithmetic is made to precede algebra, should practical geometry be made to precede theoretical geometry.

Arithmetic is not undervalued because it is inferior to algebra, nor ought practical geometry be despised because theoretical geometry is the nobler of the two.

However excellent arithmetic may be as an instrument for strengthening the intellectual powers, geometry is far more so; for as it is easier to see the relation of surface to surface and of line to line, than of one number to another, so it is easier to induce a habit of reasoning by means of geometry than it is by means of arithmetic. . . . It educates the hand to dexterity and neatness, the eye to accuracy of perception, and the judgment to the appreciation of beautiful forms. These advantages alone claim for it a place in the education of all, not excepting that of women. Had practical geometry been taught as arithmetic is taught, its value would scarcely have required insisting on.

In the Boston Latin School for a few years just preceding 1900, instruction was given by William T. Campbell in geometry under the name of observational geometry. In 1899, the results of this work were published in a textbook on observational geometry.[10] This book, and also Spencer's book, contained considerable work much like the intuitive geometry of the present day. Other geometry books were printed[11] which were similar to the two already mentioned. Early in the present century, a course in inventional geometry was given in the seventh grade of the New York public schools. Failor's *Inventional Geometry* was written in 1904 to meet the needs of this course.

[10] William T. Campbell, *Observational Geometry*. American Book Company (1899).

[11] G. A. Wentworth and Hill, *First Steps in Geometry*. Ginn and Company (1901).
I. N. Failor, *Inventional Geometry*. The Century Company (1904).

THE BEGINNINGS OF CORRELATED MATHEMATICS

The beginning of the movement for correlation in the elementary schools began about the time of the report of the Committee of Ten.[12] In fact, the committee recommended some correlation apart from the parallel courses just mentioned. It stated that familiarity with algebraic expressions and symbols, including the methods of solving simple equations, should be acquired in connection with the course in arithmetic. The early attempts at correlation were usually not successful because they were poorly planned or else were not made with a conviction that success would be attained. One of the first attempts to correlate algebra and geometry which attracted much attention was that made by Edith Long in the high school at Lincoln, Nebraska.[13] In 1902, she made the following statement:

> Two years ago, influenced by the report of the Chicago section of the American Mathematical Society, urging the simultaneous teaching of algebra and geometry, the high school at Lincoln, Nebraska, undertook the experiment of following the suggestion and introduced a course in mathematics, in which the two subjects were begun at the same time by pupils of the incoming classes.

Miss Long reported that she even went further than the recommendation and introduced physics and physical geography for the sake of a broader and more interesting application. After she had continued to teach correlated mathematics in the Lincoln High School for twelve or fifteen years, she and Professor Brenke prepared a textbook for the ninth grade and another for the tenth grade which embodied the course in mathematics as developed in this high school.[14] Miss Long reported after two years' trial that the undertaking was quite successful. Like all early attempts at correlation, however, it received severe criticism.

In the year that Miss Long began her experiment with correlated mathematics (1900), Professor J. W. A. Young published a book on *The Teaching of Mathematics in the Higher Schools of Prussia.* In the conclusion of this book he declared that it is clear that the elements of either algebra or demonstrative geometry are simpler than the more difficult parts of the other subject, that each of these subjects can be of assistance to the other, and that

[12] G. W. Myers, "Class of Content-Problems for High School Algebra," *School Science and Mathematics*, 7:19-33 (1907).

[13] Edith Long, "Correlation of Algebra, Geometry, and Physics," *Educational Review*, 24:209-11 (1902).

[14] Edith Long and W. C. Brenke, *Algebra, First Course.* The Century Company (1913).
Edith Long and W. C. Brenke, *Plane Geometry.* The Century Company (1916).

a ripe understanding of both of them is best attained if the work extends over a number of years with gradual progress to the more difficult portions as the pupil's mind develops.[15]

At the mathematical conference of the National Educational Association in the spring of 1902, Charles W. Newell, the chairman, discussed the "Correlation of Mathematical Studies in Secondary Schools." [16] After mentioning how high school mathematics was taught in one-year compartments of algebra and of geometry, he asked if there was any necessity for these artificial breaks; if it was best to complete a subject and lay it on the shelf to rust while another was taken up; and if both of them could not be taught the same day and every day.

THE PERRY MOVEMENT

About this time, a movement began in England which was destined to have considerable effect upon the teaching of secondary mathematics both in England and in America. Professor John Perry, who had been advocating reform in the teaching of mathematics for many years, presented at the meeting of the British Association in Glasgow in 1901 a rather severe criticism of the existing program of study of mathematics in England, and outlined a corrective course.[17] He said that: "Now we teach all boys mathematical philosophy that we may catch in our net the one demigod, the pure mathematician, and we do our best to ruin all the others." He advocated teaching all boys the practical applications of the subjects such as the application of the formula, the use of squared paper, numerical trigonometry, an introduction to calculus, and the use of mathematics in physics and chemistry. He was opposed to the extreme logical treatment of mathematics and in favor of teaching it in the way in which it would function best in the minds of the students. The leading mathematicians of Great Britain and some from the United States were present at the meeting in Glasgow at the time in which Professor Perry presented these ideas. Hence it is not surprising that the ideas were discussed and even partially put into practice in these two countries within the next few years.

[15] J. W. A. Young, *The Teaching of Mathematics in the Higher Schools of Prussia*, pp. 134-35. Longmans, Green and Company (1900).

[16] Charles Newell, "Correlation of Mathematical Studies in Secondary Schools." *Addresses and Proceedings of the National Education Association for 1902*, pp. 488-92.

[17] John Perry and Others, *Discussion on the Teaching of Mathematics*. Macmillan and Company, Ltd. (1901).

PROFESSOR MOORE'S ADDRESS BEFORE THE AMERICAN MATHEMATICAL SOCIETY

The most notable movement for correlated mathematics began at the University of Chicago. Before discussing this movement, we shall consider what prompted it. Professor E. H. Moore was evidently considerably influenced by this meeting because he later referred to Perry's suggestions favorably. In 1902, he gave an address before the American Mathematical Society at its ninth annual meeting, in which he said:

> As a pure mathematician, I hold as the most important suggestion of the English movement the suggestion of Perry's, just cited, that by emphasizing steadily the practical sides of mathematics, that is, arithmetic computations, mechanical drawing and graphical methods generally, in continuous relation with problems of physics and chemistry and engineering, it would be possible to give very young students a great body of the essential notions of trigonometry, analytical geometry, and the calculus. . . .
>
> The fundamental problem [in the pedagogy of elementary mathematics] is that of the unification of pure and applied mathematics. If we recognize the branching implied by the very terms "pure," "applied," we have to do with a very special case of correlation of different subjects in the curriculum, a central problem in the domain of pedagogy from the time of Herbart on. In this case, however, the fundamental solution is to be found rather by way of indirection—by arranging the curriculum so that throughout the domain of elementary mathematics the branching be not recognized. . . . The geometry must be closely connected with the numerical and literal arithmetic. . . .
>
> Engineers tell us that in the schools algebra is taught in one water-tight compartment, geometry in another, and physics in another, and that the student learns to appreciate (if ever) only very late the absolutely close connection between these different subjects and then if he credits the fraternity of teachers with knowing the closeness of this relation, he blames them most heartily for their unaccountably stupid way of teaching him. . . .
>
> Would it not be possible to organize the algebra, geometry, and physics of the secondary school into a thoroughly coherent four-year course, comparable with the strength and closeness of the course in Latin?[18]

The first movements for correlated mathematics, such as those reflected by the report of the Committee of Ten, inventional and observational geometry in this country and in Europe, Professor J. W. A. Young's remarks in his book, *The Teaching of Mathematics in the Higher Schools of Prussia*, the parallel courses in Europe, the discussion in the mathematical conference at the National Educational Association meeting, and the correlated mathematics already being taught at Lincoln, Nebraska, evidently

[18] E. H. Moore, "On the Foundations of Mathematics," Bulletin of the American Mathematical Society 9:402-24 (1903).

helped to pave the way for a more favorable reception of Professor Moore's address. No doubt these movements, as well as Perry's address, influenced Professor Moore at least to the extent that they kept the subject before his attention, and may have given him greater courage in presenting his own views. Even though the movement had already had its beginning in the United States, it had not attracted much attention. Professor Moore, being one of the leading mathematicians in this country, brought the matter of the unification of mathematics quite forcibly before the other mathematicians by his address before the Society. This address gave impetus to the movement for correlated mathematics and was the forerunner of the University of Chicago movement for the correlation of the different subjects in mathematics in the high school.

This type of address was so different from what was expected and from the views accepted by the great majority of the mathematicians present that it aroused considerable opposition. The effect was probably greater than it would have been if most of the members had agreed with the speaker. This address, together with that of Professor Perry at Glasgow, apparently created greater interest in mathematics and prompted the organization of certain active associations of teachers of mathematics. Within the next two years three important associations were formed; namely, the Association of Mathematics Teachers of New England, the Association of Mathematics Teachers of the Middle States and Maryland, and the Central Association of Science and Mathematics Teachers. Other organizations soon followed, and it was at the meetings of these associations that many improvements in the teaching of mathematics were advocated and discussed.

CORRELATED MATHEMATICS AT THE UNIVERSITY OF CHICAGO

Soon after Professor Moore's address (1902), correlated mathematics was introduced into the University High School at the University of Chicago under the direction of Professor G. W. Myers. The preliminary edition of a ninth grade textbook was published in 1906 and the second or regular edition in 1909.[19] This was probably the first modern textbook in this country to fuse algebra and geometry to any considerable extent. Approximately two-thirds of the book is algebra, including applications to physics;

[19] G. W. Myers, *First-Year Mathematics for Secondary Schools.* University of Chicago Press (1909).

a little more than one-fourth is geometry; and the remainder is arithmetic. The subjects, however, are not treated separately, but are made to serve each other in the development of the subject matter of the course. The tenth grade book was published in 1910 and is very much like the one for the ninth grade except that demonstrative geometry instead of algebra receives major emphasis.[20] In Chapters VI and VII the differences between these books and the current textbooks will be considered. Not only was a course in correlated mathematics developed by Professor Myers in the University of Chicago High School, but it was given wide publicity through discussions at the meetings of the Mathematics Section of the Central Association of Science and Mathematics Teachers, and by numerous articles published in periodicals advocating some sort of unified mathematics or discussing the movement then in progress in the University High School.

While this study is not concerned with the correlation of mathematics and physics, it should be stated in passing that quite a little interest developed about this time in the correlation of these two subjects. Frequent applications of physics are found in the Myers textbooks. Several schools gave courses in which there was considerable correlation of mathematics and physics.[21] In fact, the early movement for the correlation of the mathematical subjects was intimately related to that of the correlation of mathematics and physics.

GENERAL MATHEMATICS IN THE HORACE MANN HIGH SCHOOL FOR GIRLS

The writer has been told on good authority that general mathematics was taught in the Horace Mann High School for Girls in New York City very early in the present century. He was informed that on certain days algebra and geometry were so well fused that visitors would sometimes ask whether the class was being given a course in algebra or in geometry. No report of this experiment was printed. In the *Teachers College Record* of 1906,[22] the course of study was printed. The Horace Mann High School had recently changed from a four- to a five-year high school. The

[20] G. W. Myers, *Second-Year Mathematics for Secondary Schools.* University of Chicago Press (1910).

[21] F. L. Bishop, "Progress in the Correlation of Physics and Mathematics," *School Science and Mathematics* 5:152-59 (1905).

[22] "The Mathematics Course in the Horace Mann High School," *Teachers College Record*, 7:88-103 (1906).

course of study was as follows: first year, arithmetic and algebra; second year, elementary algebra; third year, algebra the first half-year and geometry the second half-year; fourth year, geometry the first half-year and two periods weekly of plane geometry and two of algebra during the second half-year. The only reference to the correlation of subject matter was the following: "The attempt is made to relate subjects to one another and to make them interesting and of practical value." It is easier to relate the subjects when teaching them than to do so in an orderly arranged course of study. Hence it is probable that the course of study did not reveal the type of work that was being done.

DISCUSSIONS AND REPORTS OF THE MATHEMATICS SECTION OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

Not long after the Central Association of Science and Mathematics Teachers was organized in Chicago in 1903, the Mathematics Section became interested in the reorganization of mathematics in the high schools. At the meeting of this Section on December 1, 1906, a committee of five was appointed to prepare a course of study in algebra and report back to the Section at a future meeting. This committee prepared a report the next year and published it in the official organ of the Association.[23] The committee suggested that the connection between arithmetic and algebra be made as close as possible and that mensuration problems of areas and volumes be used in the algebra course. In regard to the unification of the different mathematical subjects, the committee made the following statement:

> To bring the different branches of mathematics into closer relation and to preserve the unity of mathematical work is so much to be desired in secondary schools that the simultaneous teaching of arithmetic, algebra, and geometry would seem indispensable. By this is not meant that so many hours per week be devoted to one subject and the remaining hours to another—not parallel teaching—but the actual blending of the subjects wherever the relations are so close that such a blending will not be forced.

At the November meeting in 1907, this report was discussed and the reaction of most of the members participating in the discussion was quite favorable. A number of them, however, were leading advocates of correlated mathematics.

[23] Charles Ammerman and Others, "Preliminary Report of the Committee of the Central Association on Algebra in the Secondary Schools," *School Science and Mathematics*, 7:674-85 (1907).

So much interest was created by this report that a committee on the unification of mathematics was appointed. The preliminary report of this committee was published in *School Science and Mathematics* before the annual meeting of the Association in 1908.[24] The committee urged the necessity of embracing every opportunity for introducing into our teaching any phase of mathematics which will throw light upon the topics under consideration; of not being tied down by tradition and custom to a cut-and-dried treatment of the same; and of knowing arithmetic, algebra, geometry, and trigonometry, in their relations to one another, so that there will be no hesitation about introducing material from one subject into the teaching of another subject whenever such material is needed.

It emphasized somewhat more strongly the unification of mathematics already suggested by the algebra committee. According to the minutes of the Mathematics Section of 1908 in which this report was discussed, the reaction was again in sympathy with the unity of mathematics.[25] The committee was continued for another year, during which time it attempted to discover the work being done by other associations in the line of unification of mathematics. At the meeting in 1909, the committee reported that little or nothing had been done by other associations, but that more or less interest was manifested which was likely to result in something definite in the near future.

This report and its discussion represented the height of the movement in the Central Association for the correlation or unification of mathematics. Interest was maintained, however, in the unification of mathematics. A committee on the application of mathematics to the vocational field, another on the correlation of mathematics, and still another on the content of the course in first-year algebra gave one or more reports each in the period from 1914 to 1918. All of these were favorable to the unification of mathematics. This interest by the members of the Central Association went hand in hand with the development of correlated mathematics in the University of Chicago High School and each was a help to the other.

[24] H. E. Cobb and Others, "Preliminary Report of the Committee of the Mathematics Section of the Central Association on the Unifying of Secondary Mathematics," *School Science and Mathematics*, 8:635-44 (1908).

[25] Mabel Sykes, "Report of the Meeting of the Mathematics Section of the Central Association of Science and Mathematics Teachers, Chicago, Nov. 27, 28, 1908," *School Science and Mathematics*, 9:89-92 (1909).

HIGH SCHOOL MATHEMATICS IN THE EAST

Even though the addresses by Perry in Glasgow and Moore in Chicago were responsible for the greater interest in mathematics which caused the creation of two strong eastern associations of mathematics teachers, still these associations were very conservative in regard to the reforms advocated. About the time the Central Association was making its strongest plea for unified mathematics, a committee of the Association of the Middle States and Maryland was preparing a syllabus for a course in algebra. This syllabus was published in 1909.[26] It was pure algebra along the traditional lines. There was very little trace of the movement for unified mathematics which was so prominent in the Central Association. Certain addresses or discussions at various times showed some interest in correlated mathematics, but nothing definite was done for many years. In 1919, a committee appointed by the Association of Mathematics Teachers of New Jersey gave a report on first-year mathematics in which the following statements were made:

> In the arithmetic which stands out frequently in this syllabus there is a threefold purpose:
>
> 1. To keep alive a feeling of acquaintance with the arithmetic of the lower grades.
> 2. To make clear by the algebraic formula or equation many of the processes or problems of arithmetic, and
> 3. To gain impetus from the operations of algebra by showing their close analogy with those of arithmetic.
>
> The topics from geometry in the first place afford the best available applications of formulas, whether as numerical substitution exercises, as literal equations, or as concise symbolic expressions of verbal rules. . . .
>
> This chain of correlated ideas links up with the arithmetic of the grades and with the geometry of the following year. It is not to be forgotten that mathematics affords within itself rich territory for profitable correlations and cross-references.[27]

This syllabus represented a decided change from that reported by the committee of the Middle States and Maryland about ten years before. It should not be thought, however, that this course of study indicated what was to be done in New Jersey. No doubt

[26] Gustave Le Gras and Others, "Report of the Committee on Elementary and Intermediate Algebra of the Teachers of Mathematics of the Middle States and Maryland," *School Science and Mathematics*, 9:900-17 (1909).

[27] Arthur W. Belcher and Others, "Association of the Mathematics Teachers of New Jersey. Report of the Committee of First-Year High School Mathematics," *Mathematics Teacher* 12:61-74, 89-100 (1919).

certain teachers tried to follow it and others thought it too radical. Just as in the Central Association, such syllabi represented a desire to improve the teaching of mathematics rather than an indication of what was being done.

The situation in the New England States has been very much like that in the Middle States. Not much active interest was shown in correlated or unified mathematics in the early history of the New England Association. An experiment in general mathematics has been conducted in one or two high schools in Boston for quite a number of years. The results of this study were published in the form of a textbook by Evans and Marsh in 1916.[28] Other investigations have been made, no doubt, but none of them has attracted much attention outside of the state in which the investigation was made.

REPORT OF THE COMMITTEE OF FIFTEEN ON GEOMETRY SYLLABUS

In 1908, the American Federation of Teachers of the Mathematical and Natural Sciences authorized the appointment of a committee of fifteen on geometry syllabus. The National Education Association had already considered the advisability of such a committee and the next year approved the one already selected by the American Federation. Thus the committee's work was done under the auspices of both organizations and its recommendations reached a large number of teachers. Its membership consisted of a number of the leading mathematics teachers in the United States. A rather lengthy report was prepared by the committee and was published in 1911.[29] Although the report did not definitely advocate correlated mathematics, the statement which is quoted below indicated somewhat the attitude of the committee:

> Many of the theorems of geometry may be stated to advantage in algebraic form, thus giving definiteness and perspicuity and especially emphasizing the notion of functionality. . . . The algebraic statements are often superior to the usual statements in point of brevity and conciseness. . . .
>
> Some propositions can be proved simply and elegantly by methods involving algebra. It is somewhat usual in textbooks on geometry to give a proof of the geometric statement of such an algebraic formula as $(a + b)^2 = a^2 + b^2 + 2ab$.
>
> Some exercises for algebraic solution, such as are found in many recent texts, should find a place in any course in geometry.

[28] G. W. Evans and J. A. Marsh, *First-Year Mathematics*. Charles E. Merrill Company (1916).

[29] Herbert E. Slaught and Others, "Provisional Report of the National Committee of Fifteen on Geometry Syllabus." *School Science and Mathematics*, 11:329-55, 434-60, 509-31 (1911).

THE INTRODUCTION OF GENERAL MATHEMATICS

The crest of the wave for correlated mathematics seems to have passed by 1916 and another movement very much like it but still somewhat different seems to have begun. So much emphasis was placed upon correlation by some enthusiasts that many teachers began to react unfavorably to so much correlation. The new movement was for what is commonly called general mathematics. Someone has said quite truly that general mathematics is a corollary of correlated mathematics. Certainly correlated mathematics made the pathway easier for the new type since they are so much alike. In fact, correlated mathematics is general mathematics with the perspective placed upon the correlations of the subjects. The new general mathematics made use of the correlations when they were helpful, but the real objective was to teach the best of the mathematics to the high school pupils regardless of the amount of correlation secured. In other words, the attempt was to teach mathematics and not the relations between arithmetic, algebra, geometry, and trigonometry. It did not represent a movement to give up correlated mathematics, but to modify it in a way experience seemed to show to be a wise one.

A number of experiments have been performed in developing courses in general mathematics. In this chapter only the most significant ones will be discussed. One of these was made in the University High School of the University of Minnesota. Professor W. D. Reeve, who was at the head of the mathematics department in that high school, began developing a course in general mathematics in Minnesota in 1915. He had taught in the University High School in Chicago and was thoroughly familiar with the work done there. After several years of experimental study, Professor Reeve with the coöperation of Professor R. Schorling wrote a textbook in first-year general mathematics which was published in 1919,[30] Professor Reeve alone wrote a textbook on second-year general mathematics which was published in 1922. These two books provide for the breaking up of the compartments into which mathematics has been divided. They combine and fuse the different branches to a considerable extent, but not so much as the correlated mathematics textbooks published a few years earlier have done. Further experimental work has resulted in the recent

[30] R. Schorling and W. D. Reeve, *General Mathematics—Book I.* Ginn and Company (1919). W. D. Reeve, *General Mathematics—Book II.* Ginn and Company (1922).

publication of a new series of general mathematics books.[31] The Breslich correlated mathematics books[32] have also been revised recently in such a way that the subjects are not correlated much more than in the general mathematics books by other authors.

The work done in general mathematics at the University High School in Minneapolis stimulated the general mathematics movement in Minnesota. A considerable number of schools in the state are teaching general mathematics; others have adopted a course which is a compromise between this and the traditional mathematics type; and a number of teachers in the schools offering traditional mathematics are interested in general mathematics. One or two interesting experiments in Minnesota will be considered briefly in Chapter IV.

General mathematics, like any other attempted reform, has been accepted by some and rejected by others. Most of those who have refused to accept this new kind of mathematics in toto have adopted a compromise between this and the traditional type. The recently published textbooks and courses of study for the ninth grade are more like the general type of mathematics than the traditional algebra course of twenty-five years ago. Not so much change has been made in the plane geometry of the tenth grade, but the influence of the general mathematics movement is also manifesting itself somewhat in this grade, as will be shown in Chapter VII.

CAUSES OF THE RISE AND SUCCESS OF GENERAL MATHEMATICS

Various reasons have been given for the rise and success of general mathematics. Some of them are valid and some are not. Certain facts stand out prominently as having aided in the development and growth of this type of mathematics. Some of the most significant factors will be mentioned:

1. *Correlated Mathematics.*—The movement for correlated mathematics in the middle-western states paved the way for general mathematics when other factors were ready to contribute. Underlying the correlated mathematics movement are the influences of Perry, Moore, the parallel and fused mathematics of

[31] D. E. Smith, J. A. Foberg, and W. D. Reeve, *General High School Mathematics—Book I.* Ginn and Company (1925).
D. E. Smith, J. A. Foberg, and W. D. Reeve, *General High School Mathematics—Book II.* Ginn and Company (1926).

[32] E. R. Breslich, *Senior Mathematics—Book I.* University of Chicago Press (1928).
E. R. Breslich, *Senior Mathematics—Book II.* University of Chicago Press (1928).

Europe and the United States, and the courses developed in inventional and observational geometry.

2. *Educational Psychology and a New Philosophy of Education.*—The disciplinary idea of education and the transfer of training gradually came into disfavor in the last quarter-century and psychologists began to demand that, since there is relatively little transfer, education must be practical. This was not in accord with a logical treatment of algebra or even of demonstrative geometry. For this reason, mathematics teachers began looking for mathematical material which would be valuable to the high school pupils from a practical point of view. They became more concerned with what was real and useful than with the beautiful development of a logical structure.

3. *The Popularizing of High School Education.*—At the time correlated mathematics was being ardently advocated in the Central Association, only a relatively small proportion of the boys and girls of high school age were in school. According to Professor E. L. Thorndike,[33] in 1918, almost one child in every three children reaching their 'teens in the United States entered high school, whereas the corresponding number in 1890 was not more than one in ten. This large increase in attendance in high schools has affected the quality of students. Instead of a selected group of boys and girls of rather high intelligence being enrolled in our high schools, there is now a large number of pupils who have little more than average intelligence. Professor Thorndike has estimated that it generally requires an intelligence quotient of approximately 110 to complete the course in algebra successfully. This would eliminate about one-half of the high school pupils. Certainly the number of failures in algebra has been too high. The general mathematics course has remedied this evil to a certain extent by providing for the teaching of the easier parts of each subject first and leaving the more difficult parts for later study. General mathematics is more easily adapted to the needs of either the weak or the strong pupils than the traditional course. Hence the number of pupils who fail may be considerably reduced. At the same time, the course may be made more worth while and a better knowledge of mathematics given.

4. *The Junior High School.*—About the time that general mathematics was first being advocated, the junior high school

[33] E. L. Thorndike, *The Psychology of Algebra*, Chapter I, pp. 3, 36–37. The Macmillan Company (1923).

movement was just getting well under way. Professor L. V. Koos reported the number of junior high schools as follows:

> The handful of reorganizations that had been effected by 1910 and to which were applied the terms "junior high school" and "intermediate school" has been multiplied until they are now numbered by the thousands. Data made available by the United States Bureau of Education show that in 1925 there were 2,548 "non-four-year" high schools, these including as predominant types 879 "segregated" junior high schools and 1,389 "junior-senior" high schools (housing both junior and senior units). Numerous other communities are giving serious consideration to the proposal to effect immediate and early organization. Millions have been and are being voted to spend by some of these cities for buildings for properly housing the new institution.[34]

The new organization gave opportunity to introduce a new kind of mathematics in these three grades. Many educators have advocated a broad exploratory course in the junior high school. This has aided the cause of general mathematics. The freedom to undertake something new has also enabled those who devised the course of study in mathematics to turn to general mathematics. Most of the junior high school textbooks, even the first ones published about 1917, have been of the general mathematics type. The authors of ninth grade mathematics in the four-year high school have followed their lead to a certain extent and have introduced a modified form of general mathematics in their textbooks. The latter represent a position somewhere between the traditional type and the general mathematics type.

5. *The National Committee's Report.*—A very important factor in the general mathematics movement is the report of the National Committee. This committee was appointed by the Mathematical Association of America in 1916 and consisted of a number of the leading mathematics teachers of the United States, both in the college field and in the high school field. The nature of the report of the committee was pretty generally known in 1920 when a preliminary report was made. The final report was published in 1923.[35] The committee recommended arithmetic, intuitive geometry, algebra, numerical trigonometry, and a small amount of demonstrative geometry for the junior high school. On page 31 the report gives a full statement about ninth grade mathematics.

[34] Leonard V. Koos, *The Junior High School*, p. 8. Ginn and Company (1927).

[35] "The Reorganization of Mathematics in Secondary Education." A Report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America (1923).

In planning the work of the ninth grade under these conditions teachers and administrative officers should again be guided by the principle of giving the pupils the most valuable mathematical information and training which they are capable of receiving in this year with little reference to future courses which the pupil may or may not take. It is to be assumed that the work of this year is to be required of all pupils. Since for many this will constitute the last of their mathematical instruction, it should be so planned as to give them the widest outlook consistent with sound scholarship.

Under these conditions it would seem desirable that the work of the ninth grade should contain both algebra and geometry. It is, therefore, recommended that about two-thirds of the time be devoted to the most useful parts of algebra, including the work on numerical trigonometry, and that about one-third of the time be devoted to geometry, including the necessary informal instruction and, if feasible, the first part of demonstrative geometry.

It should be clear that owing to the greater maturity of the pupils much less time need be devoted in the ninth grade to certain topics of intuitive geometry (such as direct measurement, for example) than is desirable when dealing with children in earlier grades. Even under the conditions presupposed, pupils will be acquainted with most of the fundamental geometric forms and with the mensuration of the most important plane and solid figures. The work in geometry in the ninth grade can then properly be made to center about indirect measurement and the idea of similarity (leading to the processes of numerical trigonometry), and such geometric relations as the sum of the angles of a triangle, the Pythagorean proposition, congruence of triangles, parallel and perpendicular lines, quadrilaterals, and the more important simple constructions.

The committee was not so definite in its recommendations for the tenth grade. It offered four plans for the work of this year:

Plan A: Plane demonstrative geometry, algebra.
Plan B: Plane demonstrative geometry, solid geometry.
Plan C: Plane demonstrative geometry, trigonometry.
Plan D: Algebra, statistics, trigonometry.

Thus it is apparent that the committee was not in favor of confining the work of the tenth grade to plane demonstrative geometry alone. The chief emphasis, however, was placed upon this subject in most of the plans offered. After calling attention to the movement to break away from the older rigid division into subjects and to arrange the subject matter psychologically and pedagogically, the report continues:

There has thus developed the movement toward what are variously called "composite," "correlated," "unified," or "general" courses. The advocates of this new method of organization base their claims on the obvious and important interrelations between arithmetic, algebra, and geometry (mainly intuitive), which the student must grasp before he can gain any real insight into mathematical methods and which are inevitably obscured by a strict ad-

herence to the conception of separate "subjects." The movement has gained considerable new impetus by the growth of the junior high school, and there can be little question that the results already achieved by those who are experimenting with the new methods of organization warrant the abandonment of the extreme "water-tight compartment" method of presentation.

The newer method of organization enables the pupil to gain a broad view of the whole field of elementary mathematics early in his high school course. In view of the large number of pupils who drop out of school at the end of the eighth or the ninth school year or who for other reasons then cease their study of mathematics, this fact offers a weighty advantage over the older type of organization under which the pupils studied algebra alone during the ninth school year, to the complete exclusion of all contact with geometry.

It should be noted, however, that the specific recommendations as to content given in the next two chapters do not necessarily imply the adoption of a different type of organization of the materials of instruction. A large number of high schools will for some time continue to find it desirable to organize their courses of study in mathematics by subjects—algebra, plane geometry, etc.[36]

The report of the National Committee probably has had a greater effect upon the permanent improvement of mathematics than any other single cause. This report represented a very careful study of a number of leading mathematicians and teachers in both the college and secondary fields. So important have textbook writers considered it that scarcely a textbook in junior or senior high school mathematics has been published since 1923 in which the author has not stated that the book conforms to the recommendations of the National Committee. Some of these conformities have been rather stilted and forced, but nevertheless the report has had tremendous influence.

6. *The Work of T. Percy Nunn.*—For many years the work of T. Percy Nunn of London, although he has not been well known in America, has had considerable influence. He advocates a psychological approach to mathematics rather than a logical treatment of the subject.[37] He does not hesitate to introduce geometric formulas, to use geometric illustration in factoring such expressions as $(a + b)^2 = a^2 + 2ab + b^2$, or any other form of geometry in his teaching of algebra. He also includes numerical trigonometry as a part of the algebra course.

[36] "The Reorganization of Mathematics in Secondary Education." A Report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America (1923).

[37] T. Percy Nunn, *The Teaching of Algebra (Including Trigonometry)*, p. 19. Longmans, Green and Company, Ltd.

CHAPTER IV

RESULTS OF THE TEACHING OF GENERAL MATHEMATICS

Certain experiments and other statistical evidence tend to show the success or failure of general mathematics. The purpose of this chapter is to present a brief summary of these evidences and experiments so far as the writer has been able to collect them. Some of the data are merely suggestive and some give definite evidence.

AN EXPERIMENT IN DULUTH, MINNESOTA

In 1921, W. D. Burks of Duluth, Minnesota, at the request of the superintendent of schools, began an experiment comparing the efficiency of a two-year course in general mathematics with the traditional one-year courses of algebra and geometry.[1] He initiated his study by putting the pupils, sixty-three in number, beginning mathematics in February, in a general mathematics course. In the fall of 1921, he selected at random one hundred pupils beginning mathematics and placed them in general mathematics classes. In 1923, when the first group, which was now reduced to twenty-one through failures and withdrawals, was ready to complete the two years' work, the pupils were paired with pupils chosen from one hundred algebra-geometry pupils and equated as to intelligence, age, and quality of work done in mathematics and in their other subjects. Both groups were given a standardized algebra test, a standardized geometry test, and a test devised by the teachers of the local high school.

In discussing the advantages and disadvantages of the general mathematics group, Burks said:

> Two of the teachers of general mathematics had had several years' experience in teaching algebra and geometry but had never taught general mathematics, while the third, with as much experience in algebra and geometry, had taught only one semester of general mathematics. Moreover, these teachers soon be-

[1] W. D. Burks, "An Experiment Comparing the Efficiency of General Mathematics with Algebra and Geometry," *Mathematics Teacher*, 17:343-47 (1924).

came discouraged with the work. At times they advocated dropping it altogether and putting these pupils back into the algebra and geometry course. . . . Most of the general mathematics group had the advantage of having their teachers longer. Some had one teacher all the time, while the majority had only two. Some of the algebra-geometry group had as many as four different teachers, while only a few had the same teacher all the time. Quite a number had their freshman work in the junior high schools.

In summarizing the results of the tests given to the two groups of twenty-one pupils each, the author made the following statement:

While the general mathematics group had a median intelligence quotient two points higher and a median mathematics average 2.6 points higher, they had the same median grade in average school work. They excelled the algebra-geometry group by 6.5 points in the teachers' test and by .8 of a point in the standardized algebra test. They fell below the algebra-geometry group 1.8 points in the standardized geometry test.

The general mathematics group of one hundred was reduced to fifty-three at the end of two years. Instead of being equated with fifty-three pupils in the algebra-geometry group, they were compared with the entire group of two hundred seven. In reporting the results of the tests given to these two groups, Burks said:

The general mathematics group of fifty-three and the algebra-geometry group of two hundred seven have practically the same intelligence quotient medians, 103.5–104. The median scholarship average of both groups is identical. In mathematics the median semester grade of the general mathematics group is 80 as compared with 76 for the algebra-geometry group in geometry. The general mathematics group of fifty-three scored 29.5 in the Illinois Algebra Test as against 27 for the algebra-geometry group of two hundred seven. The former group in the Thurstone Geometry Test scored 8 as against 7 made by the latter group. Also in the teachers' test, the general mathematics group ranked 13 points higher than the algebra-geometry group.

Not only were the groups compared by testing, but they were also compared as to their enrollment and success in advanced algebra and solid geometry. The results were summarized as follows:

The median scores of the twenty-four pupils from the general mathematics group of fifty-three, who elected advanced algebra, are G.I.Q. [General Intelligence Quotient] 110; semester scholarship average 83; general mathematics grade 83.5; and advanced algebra 81.5 as compared with median scores of 107, 84, 82, and 75 respectively, of the thirty-two from the algebra-geometry group electing advanced algebra. The median scores of the thirteen pupils from the general mathematics group of fifty-three electing solid geometry are

G.I.Q. 109, scholarship average 84, general mathematics grade 80, and solid geometry 80, as compared with medians 106, 82, 77, and 77 respectively of the twenty-nine from the algebra-geometry group of two hundred seven electing solid geometry.

No better summary of this experiment can be made than that given by the author himself when he said:

> The experiment seems to show that as much or more mathematical ability is acquired in seventy weeks of general mathematics as in seventy-six weeks of algebra and geometry, for at least six weeks in general mathematics is given to trigonometry, statistics and other material not studied in algebra and geometry. In the second place, the course is much more interesting to the pupil than the old type, as shown by the fact that in Table II forty-five per cent of the general mathematics group [of fifty-three] elected advanced algebra, while but fifteen per cent of the algebra-geometry group [of two hundred seven] did so. Also in Table II, twenty-four per cent of the general mathematics group elected solid geometry, while but fourteen per cent of the algebra-geometry group did so. . . . In Table I, only one student of the algebra-geometry group [of twenty-one] elected advanced algebra, while of the general mathematics group [of twenty-one] five elected advanced algebra. Replies to a questionnaire given to the pupils of both groups as to why they elected advanced mathematics, revealed the fact that practically all of the algebra-geometry group did so because it was a part of their course, while a majority of the general mathematics group elected the advanced work because they enjoyed mathematics.

Those who were taking the advanced courses were given the traditionally arranged subject matter. The advanced algebra used was by the same authors as the elementary algebra used by the algebra-geometry group in the ninth grade. (It would be interesting to know the reactions if a third year of general mathematics had been offered to them.) This experiment is quite suggestive, but it should be supplemented by other similar experiments before definite conclusions are drawn. If the pupils knew they were being subjected to an experiment, as they probably did, their attitude toward the course may have been changed. Certainly, if Burks is correct, the attitude of the teachers was not such as to create any great enthusiasm for general mathematics.

A MASTER'S THESIS AT THE UNIVERSITY OF CHICAGO

Two studies were made by L. V. Bowyer in 1923 and the results recorded in an unpublished Master's thesis at the University of Chicago.[2] The first was a comparison of the results of algebra

[2] L. V. Bowyer, "A Study of the Results of High-School Instruction in Correlated Mathematics." An unpublished Master's thesis at the University of Chicago (1923).

and geometry tests given to classes in both correlated and traditional mathematics completing one year, two years, and three years of mathematics in the high school. Either two or three classes were selected in which pupils had had one year, two years, and three years respectively of traditional mathematics, and a

TABLE 1

COMPOSITE ALGEBRA SCORES OF CLASSES IN ORDINARY AND CORRELATED MATHEMATICS

Type of Instruction	1st Year	2nd Year	3rd Year
Ordinary Mathematics ...	25.9	23.6	31.4
Correlated Mathematics ..	25.1	25.9	37.8

corresponding number of classes in which pupils had had one year, two years, and three years of correlated mathematics. An attempt was made to select classes in the two types of mathematics in which the pupils were as nearly equivalent as possible in ability and those which were taught by equally skillful teachers. This involved considerable difficulty, and it was found necessary to depend upon subjective judgment. Bowyer reported that the classes were apparently fairly comparable. The summarized results of the testing in algebra are given in Table 1 and the results for geometry are given in Table 2.

TABLE 2

COMPOSITE GEOMETRY SCORES OF CLASSES IN THE SECOND AND THIRD YEARS OF ORDINARY AND CORRELATED MATHEMATICS

Type of Instruction	2nd Year	3rd Year
Ordinary Mathematics	27.65	31.22
Correlated Mathematics	28.99	33.74

Table 1 shows that the median score in algebra for first-year classes in ordinary mathematics is slightly higher than for those in correlated mathematics. This is to be expected since Breslich's

First-Year Mathematics was used as a textbook for the classes in mathematics. This book devoted considerable space to geometry; and even though much of it is correlated with algebra, the algebra necessarily suffered somewhat. If a test had been given in geometry, it is reasonable to suppose that the correlated mathematics group would have excelled the other. In fact, it was thought unfair to those taking the ordinary course to give a geometry test in the first year. After the first year, the advantage in algebra of correlated mathematics becomes more apparent each year. Table 2 reveals a superiority in geometry of the correlated mathematics group over the other in both the second and third years, that of the third year being the greater.

In this study, correlated mathematics may have had the advantage of being taught to the experimental groups. The enthusiasm created by an experiment seems to carry over into making the work a greater success. This was counterbalanced by the fact that this kind of mathematics was not new in the schools tested and hence the enthusiasm which accompanies a new movement was not to be found.

The other study conducted by Bowyer is a comparison of the marks made by students in college who were graduated from the University of Chicago High School where correlated mathematics had been taught to them. This investigation had to do with the marks made both in the University of Chicago and in other colleges. The comparison of the marks of students in the University of Chicago was made in terms of "evaluating ratios." The evaluating ratio for a particular student was secured by dividing the student's grade points in mathematics by the average grade points of the class. Thus an evaluating ratio of 1 indicates that the pupil's mark in mathematics is the average of the class, and the amount above or below 1 indicates how much his work is above or below the average of the class. The results are summarized as follows:

> The average of the evaluating ratios for the last three years of graduates of two of the large Chicago high schools who entered the University of Chicago, was for mathematics 1.005 and for other courses 1.05. For graduates of the University High School similar averages were for mathematics 1.12 and for other courses 1.09. This would indicate that, in general, graduates of the University High School who enter the University of Chicago do only slightly better work in courses other than mathematics than do graduates of the large city schools, but in mathematics they do considerably better.

A similar comparison was made of the marks made in college mathematics and in other subjects by graduates of the University High School who entered various colleges and universities other than the University of Chicago. The tabulation of the marks shows that from 1916 to 1922 inclusive, 15 students made lower marks in mathematics than in other subjects; 15, the same; and 45, higher marks. There would seem to be four possible reasons for the greater number receiving higher marks in mathematics, namely: (1) the average marks in mathematics in these colleges and universities may have been higher than the marks in other subjects; (2) it may have been due to chance and if a sufficiently large number of students were available the results might be quite different; (3) the students in the University High School may have received better training in mathematics than in other subjects; and (4) correlated mathematics may give better preparation for college mathematics than the ordinary type. No comparable data are available as to the average relative marking of these colleges in mathematics and in other subjects. It does not seem to be the usual practice of mathematics teachers to mark students higher than do teachers of other subjects. Since the seventy-five students were distributed in twenty-two different colleges, it cannot be assumed that the results were greatly influenced by peculiarities in the marks given in any one college. Neither is it probable that the element of chance operated to such an extent as to make the number three to one in favor of correlated mathematics if there is no actual difference between the two, even though only seventy-five students were considered. Therefore it seems reasonable to assume that either the teaching of mathematics in the University High School was better than in other subjects or that the superior results were due to better preparation for college being given by correlated mathematics than by ordinary mathematics.

A DOCTOR'S DISSERTATION AT THE UNIVERSITY OF CHICAGO

In 1926, Professor E. R. Breslich reported the results of a testing program in the University High School in Chicago.[3] The tests were given in the junior high school connected with the University where Breslich's *Junior Mathematics* was taught and in other elementary and high schools where the ordinary courses in mathe-

[3] E. R. Breslich, "A Critical Examination and Experimental Reconstruction of Secondary Mathematics." An unpublished Doctor's dissertation at the University of Chicago (1926).

matics were taught. The tests given to the seventh and eighth grades were confined to tests of arithmetical ability. Before the results are considered, a brief explanation of the difference between the Breslich books and the ordinary mathematics books taught in the seventh and eighth grades will be given. A comparative study of textbooks made by the writer is reported in Chapters V, VI, and VII. It was found that in the Breslich books for the seventh and eighth grades, the work is divided almost equally among arithmetic, algebra, and intuitive geometry, with a small amount of statistical graphs and trigonometry. The arithmetic mean of the different branches of mathematics in the ordinary seventh and eighth grade arithmetics published recently was found to be 80.4 per cent arithmetic, 5.5 per cent algebra, 10 per cent intuitive geometry, and 4.1 per cent statistical graphs. It would seem, therefore, that the Breslich books would produce poorer results in arithmetic and better results in algebra and intuitive geometry than the ordinary arithmetics. Professor Breslich reported, however, that his investigation showed that "Classes

TABLE 3

MEDIANS FOR TESTS IN THE FUNDAMENTALS OF ALGEBRA

Year	Class	Grade	No. of Pupils	Test I	Test II	Test III
1926	XVI	X	34	4.5	9.0	6.2
1926	XVII	X	28	4.5	10.0	4.8
1926	XVIII	IX	19	5.1	7.9	5.1
1926	XIX	IX	27	3.9	8.7	8.4

Explanation: Classes XVI and XVII are tenth grade classes and have completed courses in first-year algebra and plane geometry. Both are in schools which rank among the best in the city of Chicago. Class XVII was designated by the teacher as an average class. Class XVI seems to be better than an average class. Classes XVIII and XIX are ninth grade classes in the School of Education.

having studied the new courses (the Breslich books) show superiority in arithmetic over classes that have devoted all of the seventh and eighth grade time to arithmetic." The seventh and eighth grade classes were tested only in arithmetic. If they had been tested in algebra and intuitive geometry, they would probably have shown superiority over classes which had had comparatively no training in these subjects.

In classes above the eighth grade, three algebra tests were given as follows: Test I—algebraic addition and subtraction; Test II—algebraic multiplication and division; and Test III—equations. The results of these tests are presented in Table 3.

According to the data of Table 3 there was not much difference in the ability of the two ninth grade classes having had correlated (or general) mathematics and the two classes having had the ordinary course in algebra. A similar study was made in regard to geometry. Tests were devised to measure the fundamentals of geometry such as are commonly taught in the junior high school. Four tests were constructed as follows: Test I—geometric terms; Test II—geometric principles and formulas; Test III—measuring and drawing; and Test IV—equations in geometric problems. Table 4 gives the median scores for the classes tested.

TABLE 4

MEDIANS FOR TESTS IN THE FUNDAMENTALS OF GEOMETRY

Year	Class	Grade	No. of Pupils	Test I	Test II	Test III	Test IV
1926	XVI	X	35	13.3	12.0	4.6	3.1
1926	XVII	X	27	12.6	6.1	4.4	1.9
1926	XVIII	IX	19	12.5	9.8	4.4	3.4
1926	XIX	IX	27	11.1	8.6	4.4	3.3

Explanation: These classes are the same as the ones tested in algebra. See explanation for Table 3.

The results of the geometry tests did not reveal a superiority of either group. The ninth grade pupils in the junior high school tested, on the average, about the same as the tenth grade geometry class. These tests in algebra and geometry indicate that pupils who have had general mathematics in the junior high school will have as good a knowledge of algebra and the fundamentals of geometry (not demonstrative) as those who have completed the traditional courses of algebra and geometry. Professor Breslich summarized the results as follows:

When tested in the fundamentals of algebra and geometry, ninth grade pupils who have studied the new course compare favorably with the tenth grade pupils who have studied first-year algebra and plane geometry. Pupils who have studied the correlated type of mathematics in the University High

School succeed in the study of college mathematics better than the average student in college. Time is saved in the new course without loss to the learner.

STUDENTS' MARKS AT THE UNIVERSITY OF MINNESOTA

1. *Data Collected by W. D. Reeve.*—In 1922, Professor W. D. Reeve, then of the University High School of the University of Minnesota, reported as follows:

It may be of interest here to say that of all our University High School students who have had at least two years of work in general mathematics, and who have subsequently taken further courses in the University, 13.2 per cent have received marks of A; 31.6 per cent marks of B; 34.2 per cent marks of C; 13.2 per cent marks of D; and 7.8 per cent marks of F. Only two students have failed and one of these was one of our honor students at graduation and the other stood at the bottom of the class. Moreover, of those who did not have at least two years of work in general mathematics, *i.e.*, those who had their mathematics before they came to us, and who have taken further mathematics in the University, no one received a mark of A; 11.7 per cent received B; 25.6 per cent received C; 39.5 per cent received D; and 23.2 per cent were given a mark of failure.[4]

2. *Data Collected by W. H. Bussey.*—Professor W. H. Bussey, of the University of Minnesota, gave some statistics in 1925, of which the following are worth noting in this connection:

There were 484 students in the freshman mathematics classes in the College of Science, Literature, and Arts of the University of Minnesota, in the fall quarter of this college year. I have classified them in three groups as follows:

	Number of Students	The nature of the student's first two years of high school mathematics
Group I	410	Algebra, 1 year; Geometry, 1 year
Group II	22	Unified Mathematics, 2 years
Group III	52	Unified Mathematics, 1 year; Geometry, 1 year

Some of the students in each of these groups studied more mathematics while they were in high school. This classification refers only to the nature of the first two years' work.

The 22 students in Group II did fully as well in college mathematics as those in Group I. The number 22 is relatively small, and therefore it is not wise to draw any general conclusion; but the figures are encouraging to the advocates of general mathematics.

[4] W. D. Reeve, "The Case for General Mathematics," *Mathematics Teacher*, 15:381-91 (1922).

The 52 students in Group III did much worse in college mathematics than those in Groups I and II. This is true even of those who had higher algebra in high school after they had finished one year of Unified Mathematics and one year of Geometry. As a group, the students in Group III had a fundamental weakness in Algebra, due to the makeshift combination of Unified Mathematics and Geometry.

3. *Data Collected by Clarence McCormick.*—The writer made an investigation of the marks of students taking their first term of college mathematics in the University of Minnesota during the year 1927–28. The students were separated into three classes as follows: first, those whose first two years of high school mathematics was general mathematics; second, those whose first year was general mathematics and whose second year was plane geometry; and third, those whose first year was algebra and whose second year was plane geometry. There were 201 in the first group, 14 in the second group, and 1,174 in the third group. A statistical report of the marks is given in Table 5.

TABLE 5

NUMBER OF STUDENTS RECEIVING DIFFERENT MARKS IN THE FIRST TERM OF COLLEGE MATHEMATICS AT THE UNIVERSITY OF MINNESOTA, 1927–1928

Mark Received*	GROUP I 2 years of General Mathematics	GROUP II 1 year of General Mathematics and 1 of Geometry	GROUP III 1 year of Algebra and 1 of Geometry
A	18	0	72
B	24	5	157
C	64	3	376
D	47	2	307
E	4	0	32
I	2	0	6
F	42	4	224
Total	201	14	1,174

*A, B, C, and D are passing marks; E is a condition; I is an incomplete; and F a failure.

These data are more easily interpreted when expressed in percentages, as in Table 6. The number of students (14) in Group II was too small to give any reliable conclusion about the effect of one year of general mathematics followed by a year of plane

geometry upon the marks made in college mathematics. In Professor Bussey's study, there were 52 students. The conclusions he reached should be much more reliable than the ones derived from only 14 students. There is very little difference between Group I and Group III. The general mathematics group (Group I) has a somewhat larger per cent of students making A's and a slightly larger per cent of failures than the group (Group III) with two years of traditional mathematics in the first two years of high school. The median mark for Group I is slightly higher

TABLE 6

PER CENT OF STUDENTS RECEIVING DIFFERENT MARKS IN THE FIRST TERM OF COLLEGE MATHEMATICS AT THE UNIVERSITY OF MINNESOTA, 1927–1928

Mark Received*	GROUP I 2 years of General Mathematics	GROUP II 1 year of General Mathematics and 1 of Geometry	GROUP III 1 year of Algebra and 1 of Geometry
A	9.0	0.0	6.1
B	11.9	35.7	13.4
C	31.8	21.4	32.0
D	23.4	14.3	26.1
E	2.0	0.0	2.7
I	1.0	0.0	0.5
F	20.9	28.6	19.1
Total	100.0	100.0	99.9

* See note to Table 5.

than that for Group III. This study seems to verify the results which Professor Bussey found in 1925. There is not enough difference in results in this study, however, to be significant.

ENROLLMENT IN ELECTIVE COURSES

Another way to investigate the success of general mathematics is to determine the relative enrollment in elective courses in mathematics of those pupils who have had each type of mathematics in the ninth and tenth grades. Other things being equal, this would show the comparative amount of interest created by the two types.

1. *Enrollment in the University High School in Chicago.*—In 1916, Professor G. W. Myers reported that mathematics was required in only the ninth grade of the University High School where correlated mathematics had been taught for a number of years. In the other grades, out of 295 students not required to take mathematics, 225 elected mathematics courses.[5] This means that about three out of every four in the upper three grades elected mathematics. The large number of pupils who chose a mathematics course might have been due to any one (or more) of a number of reasons; namely: (1) the University High School is essentially a college preparatory school in close connection with the University of Chicago and hence the pupils took mathematics to prepare for college entrance; (2) the pupils may have represented a more highly selective group than the average high school and one which would be particularly interested in mathematics; (3) the teachers of mathematics may have been better able to sell their subject to the pupils than the teachers in other departments; and (4) general mathematics may have been intrinsically more interesting than the other type. Professor Myers would have us believe that the last-named reason was the one which was effective in bringing about a large enrollment in the department.

2. *Enrollment in the English High School in Boston.*—The English High School in Boston is not connected with any college and is perhaps typical of the large city school. General mathematics has been taught in the ninth and tenth grades of this school for many years. In 1923, L. A. McCoy, a teacher in the school, stated:

> In our large general high school where mathematics is purely an elective, about 550 elect it the first year, the same number the second year, 350 the third and about 200 the fourth. . . . Almost half of our senior class elect fourth year mathematics and less than half of these are aiming for college. More than two-thirds of our junior class elect M3, mostly plane and solid geometry, and when that number of students elect solid geometry, something a little bit different has happened in their study of plane geometry.[6]

3. *Enrollment in the Eleventh and Twelfth Grades in Forty-Five High Schools.*—The writer has attempted to find evidence of the relative merits of the two kinds of mathematics in inducing pupils to continue with elective mathematics in the eleventh and twelfth

[5] G. W. Myers, "Educational Movements and General Mathematics," *School Science and Mathematics*, 16:97-104 (1916).

[6] L. A. McCoy, "Advantages of a General Course in Mathematics for the First Two Years in High School," *Mathematics Teacher*, 16:421-24 (1923).

grades. Inquiries were sent to one hundred thirty-nine high schools. An attempt was made to pair the schools offering traditional mathematics with those offering general mathematics as to number and size between the two types of mathematics. After replies were received, however, it was found that more than one-half of the schools from which information was requested were offering traditional mathematics. The part of the questionnaire which is applicable here is as follows:

QUESTIONNAIRE

There are two plans commonly used for organizing the subject matter of mathematics in the four-year high school. We shall designate these by T and G as follows:

T: Traditional arrangement of mathematics consisting of a year of algebra, then a year of plane geometry or algebra, and so on, with little or no attempt to fuse or unite the different parts of mathematics during the year.

G: General, correlated, or unified mathematics in which the different parts of the subject are fused or united in some way in the year's work. The ninth grade course, for example, will generally be more than one-half algebra, with a considerable portion of other parts of mathematics such as intuitive geometry, arithmetic, algebra, trigonometry, and so on, included.

Indicate by T or G (or by a question mark in case you do not know) the kind of a course offered in your high school and the name of the textbook and the author for the years indicated below:

Grade and Year	Kind of Course	Name of Textbook	Author
Ninth Grade 1925–26			
1926–27			
1927–28			
1928–29			
Tenth Grade 1926–27			
1927–28			
1928–29			

What is the total enrollment for this semester in the mathematics classes in the Ninth grade?.....Tenth grade?.....Eleventh grade?.....Twelfth grade?.....

What is the total high school enrollment for this semester in the Ninth grade?.....Tenth grade?.....Eleventh grade?..... Twelfth grade?....

Is mathematics required in the Ninth grade?.....Tenth grade?..... Eleventh grade?.....Twelfth grade?.....

Some of the schools did not reply to the questionnaire; some offered both types of mathematics and reported them together; others required mathematics in the eleventh grade; and the replies from still others were incomplete. None of the incomplete returns could be used for comparison. This left only about one-third of

TABLE 7

SCHOOLS IN WHICH GENERAL MATHEMATICS WAS TAUGHT IN THE NINTH AND TENTH GRADES

School Number	Is Mathematics Required?		Eleventh Grade			Twelfth Grade		
	Ninth Grade	Tenth Grade	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.
1	Yes	Yes	22	22	100.0	21	17	81.0
2	Yes	No	67	26	38.8	59	16	27.1
3	No	No	800	350	43.8	450	80	17.8
4	Yes	No	33	5	15.2	28	11	39.3
5	Yes	Yes	23	5	21.7	15	3	20.0
6	Yes	No	14	2	14.3	12	3	25.0
7	Yes	Yes	458	96	21.0	395	10	2.5
8	No	No	80	16	20.0	(43)	0	....
9	Yes	Yes	(79)	0	.....	79	15	19.0
10	Yes	Yes	216	28	13.0	(209)	0	....
11	Yes	Yes	70	4	5.7	63	2	3.2
12	No	No	74	4	5.4	(55)	0	...
13	Yes	Yes	(36)	0	.....	(44)	0	....
14	No	No	....	...	.....	260	64	24.6
Total* ..			1857 (1972)	558	30.0 (28.3)	1382 (1733)	221	16.0 (12.8)

* The numbers in parentheses represent the enrollments in schools in which mathematics was not offered to students in the grade in question. They are not included in the first totals but are included in the totals which are enclosed in parentheses.

the schools to which questionnaires were sent. Those remaining were from forty-five schools in twelve different states. The information received has been tabulated for the eleventh and twelfth grades and the results are given in Tables 7, 8, and 9. In making this tabulation, it was assumed that eleventh grade pupils had had the first year of mathematics two years before the report was made and second-year mathematics one year before. It was likewise assumed that twelfth grade pupils had had their first two years of mathematics the year before the eleventh grade pupils. The writer used this assumption as a basis for ascertaining what kind of mathematics had been given to the pupils. The results obtained by this procedure will probably not differ much from the current results, since for most high school graduates the number of years spent in high school is four.

In seven of the forty-five schools, no mathematics classes were reported in the eleventh grade, and in eleven schools no classes were reported in the twelfth grade. According to the reports from

TABLE 8

SCHOOLS IN WHICH GENERAL MATHEMATICS WAS TAUGHT IN THE NINTH GRADE AND TRADITIONAL MATHEMATICS IN THE TENTH GRADE

School Number	Is Mathematics Required?		Eleventh Grade			Twelfth Grade		
	Ninth Grade	Tenth Grade	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.
14	No	No	(293)	0	....	....	...	....
15	No	No	537	147	27.4	384	83	21.6
16	Yes	No	374	114	30.5	307	76	24.8
17	No	No	220	40	18.2	239	40	16.7
18	No	No	372	74	19.9	317	41	12.9
19	No	No	548	100	18.2	373	50	13.4
20	No	No	448	63	14.1	276	30	10.9
21	No	No	(589)	0	....	471	111	23.6
22	No	No	199	20	10.1	175	14	8.0
23	Yes	No	(58)	0	....	(42)	0	....
Total* ..			2698 (3638)	558	20.7 (15.3)	2542 (2584)	445	17.5 (17.2)

* See note to Table 7.

schools in which there were advanced classes, it would seem that in almost all of these schools some pupils would have elected a course in mathematics if one had been offered. For this reason, the enrollments of the schools in the grades in which no classes were reported, were omitted from the totals. The figures for each of these enrollments are placed in parentheses in the tables and the totals including them are placed in parentheses under the

TABLE 9

SCHOOLS IN WHICH TRADITIONAL MATHEMATICS WAS TAUGHT IN THE NINTH AND TENTH GRADES

School Number	Is Mathematics Required?		Eleventh Grade			Twelfth Grade		
	Ninth Grade	Tenth Grade	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.	Total Enrollment*	Total Enrolled in Math.	Per Cent Enrolled in Math.
24	Yes	No	81	28	34.6	51	14	27.5
25	Yes	No	650	282	43.4	435	54	12.4
26	Yes	Yes	285	86	30.2	345	87	25.2
27	Yes	No	17	9	52.9	(18)	0	....
28	Yes	Yes	398	130	32.7	372	60	16.1
29	Yes	Yes	175	60	34.3	148	20	13.5
30	Yes	Yes	359	134	37.3	291	25	8.6
31	Yes	Yes	425	75	17.6	275	75	27.3
32	Yes	Yes	43	17	39.5	(32)	0	....
33	Yes	Yes	406	120	29.6	335	27	8.1
34	Yes	Yes	612	150	24.5	421	50	11.9
35	Yes	Yes	93	32	34.4	(94)	0	....
36	No	No	38	12	31.6	(29)	0	....
37	No	No	73	12	16.4	64	9	14.1
38	No	No	237	30	12.7	196	30	15.3
39	Yes	Yes	40	5	12.5	46	7	15.2
40	No	No	179	22	12.3	128	16	12.5
41	Yes	No	44	4	9.1	37	3	8.1
42	Yes	No	60	4	6.7	48	5	10.4
43	Yes	Yes	160	12	7.5	122	9	7.4
44	No	No	(26)	0	....	(26)	0	....
45	Yes	No	(17)	0	....	(12)	0	....
Total* ..			4375 (4418)	1224	28.0 (27.7)	3314 (3525)	491	14.8 (13.9)

* See note to Table 7.

regular totals. The reader may then accept the result that he believes to be most valid. The writer believes that schools in which the pupils were not permitted to elect mathematics should not be included in the totals. Hence he will use in this discussion the results not in parentheses.

The tables show that mathematics was elected in the eleventh grade by 30 per cent of the pupils with two years of general mathematics, by 28 per cent of those with two years' traditional mathematics and by about 21 per cent of those with one year of general mathematics and one year of plane geometry. The per cents for these three groups in the same order for the twelfth grade are 16, 14.8, and 17.5. In some of the schools the eleventh and twelfth grade classes were thrown together and the numbers reported to be enrolled in each of the two grades were only an estimate. Therefore a more reliable result would be obtained by putting the two grades together and tabulating the results for the combined grades. The combined totals from such procedure are given in Table 10.

TABLE 10

COMBINED TOTALS OF PUPILS ELECTING MATHEMATICS IN THE ELEVENTH AND TWELFTH GRADES

Type of Mathematics	Enrollment	Enrolled in Mathematics	Per Cent in Mathematics
Two Years of General	3239	779	24.1
One Year of Each Type ...	5240	1013	19.3
Two Years of Traditional ..	7689	1715	22.2

Table 10 shows that the pupils who have had two years of general mathematics are more likely to take an elective course in mathematics in the eleventh or twelfth grade than those who have had two years of traditional mathematics or one year of general and one year of traditional mathematics. It also shows that those who have had one year of general mathematics and one year of plane geometry are least likely to take advanced high school mathematics than either of the other two groups.

4. *Enrollment of University of Minnesota Freshmen.*—The writer made an investigation of the enrollment of freshmen in courses in mathematics in the College of Science, Literature, and

the Arts in the University of Minnesota during the year 1927–28. Mathematics is not required in this college. The total number of freshmen was 1,849. The tabulated results are as follows:

A. Total number whose first two years of high school mathematics were in general mathematics 350
   Number who elected mathematics 70
   Number who did not elect mathematics 280
B. Total number whose first two years of high school mathematics were one year of general mathematics and one year of plane geometry 8
   Number who elected mathematics 4
   Number who did not elect mathematics 4
C. Total number whose first two years of high school mathematics were one year of algebra and one year of plane geometry 1,491
   Number who elected mathematics 323
   Number who did not elect mathematics 1,168

In order to find the relative interest created in college freshman mathematics by the different types of mathematics in the first two years of high school, it would be helpful to determine the per cent of each group of students who elect college mathematics. Since only eight students comprise the group with one year of each type, this group will be omitted from further consideration. The per cents for the other two groups are as follows:

A. Per cent of students taking college freshman mathematics whose first two years in high school mathematics were general mathematics 20.0
B. Per cent of students taking college freshman mathematics whose first year of high school mathematics was algebra and whose second year was plane geometry 21.7

This result is just the opposite to that obtained in the elective courses in the eleventh and twelfth grades in high school. The writer was somewhat puzzled as to the reason for this until after the investigation of the opinions of teachers, recorded in Chapter IX, was made. He found that there is a prevailing opinion among high school teachers that general mathematics is not a suitable course as a preparation for college mathematics. It is quite natural that this notion should be passed on to the pupils. Therefore college students who have had general mathematics have quite likely been told that college mathematics is different from the kind they have had and that they will be handicapped if they continue the subject. Thus the number who elect mathematics

has probably been reduced somewhat from what it would have been if the opinions of the teachers had been different. It is impossible to predict what the relative percentages would be if the experiment could be controlled so that all other factors would be equal. It is apparent that it would be much more favorable to general mathematics than in this investigation. In fact, it was not uncommon to see expressions of opinion that general mathematics was not a suitable preparation for third and fourth years of high school mathematics. Hence the teachers' opinions may have reacted somewhat against the election of mathematics in the eleventh and twelfth grades. It would be interesting to compare this conclusion with the reports of whether mathematics was required or not in the ninth and tenth grades in the high schools investigated. Tables 7, 8, and 9 show that no mathematics is required in the high schools by 28.6 per cent of the schools offering the first two years of general mathematics, by 80 per cent of those offering a year of general mathematics and a year of plane geometry, and by only 22.7 per cent of those offering a year of algebra and a year of plane geometry. Most colleges require mathematics for entrance and hence the schools preparing for college entrance are most likely to require two years of mathematics. This difference in procedure is almost equivalent in effect to saying to the pupils that general mathematics is not a suitable preparation for college mathematics. Such assumption is false, as has already been shown.

GENERAL CONCLUSIONS

The evidence collected in any one of these studies is far from being reliable. When the results of all of them are thrown together and viewed as a whole, they seem much more significant. The conclusions may be summarized as follows:

1. There is not much difference in algebraic ability of pupils who have finished either one year of elementary algebra or one year of general mathematics in the high school. The evidence is slightly in favor of the algebra group, however. Bowyer found that the algebra group tested 0.8 of a point higher than the general (correlated) mathematics group. This difference is so small that it could easily be due to chance. Professor Breslich found that the ninth grade general (correlated) mathematics group tested about the same as the tenth grade algebra-geometry group. This evidence is also slightly in favor of ninth grade algebra because

the ability of pupils in algebra is probably not so great when they have completed the course in plane geometry as at the end of the year's work in algebra.

2. The pupils taking general mathematics tested about the same in the fundamentals of geometry after one year in high school as the pupils having had one year of algebra and one year of plane geometry. This is shown by Professor Breslich's experiments. It would seem that, if both groups were tested after only one year of high school work, the general mathematics group would test much higher because of the fact that the algebra classes do not receive as much training in geometry. There is, however, no experimental evidence to verify this supposition.

3. In the experiments recorded, the pupils with two years of general mathematics in the high school excelled those with one year of algebra and one year of plane geometry. Two comparisons were made by Burks in Duluth and one was made by Bowyer in Chicago. In two out of the three comparisons, the pupils with general mathematics surpassed those of the algebra-geometry groups in the demonstrative geometry tests. The group of general mathematics pupils that tested lower was much the smallest one of the three, being a class of only twenty-one.

4. In the only experiment comparing two groups of third-year pupils, the group having had general (correlated) mathematics was considerably better in both algebra and geometry than the group having had three years of traditional mathematics. This experiment was made by Bowyer.

5. The pupils finishing plane geometry did not test as high in algebra as those just completing the year of algebra, while the pupils in general mathematics improved in both algebra and geometry each year.

6. The college students who have had two years of general mathematics in the high school seem to make grades in mathematics at least as high as, and probably higher than, the students who have had a year of algebra and a year of plane geometry. This is shown by both Professor Bussey's study and the writer's study of marks at the University of Minnesota and is indicated by Bowyer's study at the University of Chicago.

7. General mathematics apparently has created a liking for mathematics to such an extent that a somewhat greater per cent of pupils who have had general mathematics will elect mathematics

in the eleventh and twelfth grades than will those who have had one year of algebra and one year of plane geometry. This is indicated by two different studies: one being an investigation made by Burks in the high school at Duluth, and the other a collection of data by the writer from a number of high schools.

8. One year of general mathematics and one year of plane geometry does not create as much interest in advanced high school mathematics as two years of either type. This was revealed in an investigation by the writer. The results may have been influenced by the fact that such courses were offered for the most part in schools in which the first two years of mathematics were elective, and thus the pupils were led to believe that the study of mathematics is not essential.

9. According to Professor Bussey's study, one year of general mathematics and one year of plane geometry is not as good preparation for college as either a year of algebra and a year of geometry or two years of general mathematics.

10. It is uncertain which type of mathematics creates greater interest in college mathematics. In a study made by the writer, a slightly greater per cent of college students with two years of traditional mathematics elected freshman mathematics than those with two years of general mathematics. The writer believes, however, that this was probably due to the fact that many teachers think that general mathematics is suitable only for those pupils who do not intend to continue work in the subject. Students who take general mathematics are therefore led to believe that they should not take advanced work in the subject.

In considering these conclusions, the reader should bear in mind that most of the tests given were in either algebra or geometry alone and did not test some of the things that the general mathematics enthusiasts claim for their subject. If these tests were unfair to either group, it was the general mathematics group. For this reason the writer believes that if more can be done to test the appreciation of boys and girls for mathematics, the case of general mathematics would be more firmly established or its value discredited. The writer, therefore, attempted two of the studies recorded in this chapter, hoping to contribute a little toward testing the interest and appreciation of mathematics created by general mathematics. The experiment was handicapped by the constant factors which tended to vitiate the results.

CHAPTER V

A CRITICAL STUDY OF SEVENTH AND EIGHTH GRADE TEXTBOOKS

COMMON MISUNDERSTANDING OF GENERAL MATHEMATICS

There has been a great deal of misunderstanding of the general mathematics movement among teachers of the subject. Even those better informed than the average have often wrongly interpreted the whole movement. Many teachers would call any sort of mathematics textbook "general mathematics" if it were so labeled. Similarily, they would consider a textbook "traditional" if it were called algebra or plane geometry. Some, no doubt, would object to the term "traditional," in designating the algebra and geometry courses, because there was a time when general mathematics was the type in common use. It is the opinion of the writer that one hundred twenty-five (or more) years is sufficient time for a subject to become traditional from the point of view of the present generation. This chapter reports an examination of textbooks of the seventh and eighth grade mathematics, and the next two chapters report similar studies of textbooks for the ninth and tenth grades.

PROBLEMS FOR INVESTIGATION

Before proceeding to the study of textbooks, it was necessary to decide just what problems were to be investigated. Naturally, since this study has to do with general mathematics, one would wish to know the amount of each branch of mathematics in textbooks at different periods of time. In this way, the drift toward or away from general mathematics could be determined. Such a study would determine to a certain extent the direction of the present trend. General mathematics attempts to fuse or correlate the branches to the extent that such fusion is thought to be natural. Hence this study should show the amount of correlation indicated by the books at different periods of time.

Since there have been certain well-defined movements in the

development of general mathematics, it would be interesting to determine the effect of each of these movements. The movement for correlated mathematics did not become important until after 1902. This movement began to decline by 1916 and to be replaced by a general mathematics in which correlation was not stressed so much. Hence an investigation of textbooks published about 1900 and others about 1915 would reveal the effects of the correlated mathematics movement. The movement for general mathematics with less emphasis on correlation was well under way by 1920 and the National Committee's report was published in 1923. Another factor that has helped general mathematics very materially is the junior high school movement. This did not become important until after 1915. Hence an investigation of textbooks of quite recent date in comparison with those of 1915 would reveal the influence of these factors.

SEVENTH AND EIGHTH GRADE TEXTBOOKS EXAMINED

The early arithmetics were written to cover the material to be taught in arithmetic without reference to the grades in which the books were to be used. This was true to a considerable extent as late as 1900. Hence it was rather difficult to select textbooks which represented only the seventh and eighth grade work. After considerable searching, the writer found five books published between 1897 and 1902 which apparently represented the work of these grades fairly well. They were written for the seventh and eighth grades or for the grammar schools. The Teachers College Library at Columbia University has six seventh and eighth grade textbooks published between 1913 and 1916, and eleven since 1925. All of these, twenty-two in number, were examined by the writer.

These arithmetic books were written primarily for the seventh and eighth grades in schools organized on the 8–4 plan. Series of books have also been written for the junior high school. In 1928, there were fifteen or sixteen such series on the market. Eleven were published since 1923. All but one of these eleven have been examined. The one which was omitted treats arithmetic as the chief topic in the ninth grade as well as in the seventh and eighth grades. Other arithmetics, written for the ninth grade in schools on the 8-4 plan, have not been considered. For this reason, it was thought best to omit this series of books likewise. The results of the investigation of textbooks for the first two years of junior

high school are given in this chapter and those for the ninth grade are left for the next chapter. In order to obtain an idea of the unity of the junior high school series of books, the two chapters must be considered together.

METHOD OF PROCEDURE

A study of the general mathematics in textbooks requires information about the amount of each of the various branches of mathematics contained in them. There are different ways in which this investigation might be conducted. Perhaps the easiest way would be that in which the portions of the textbook devoted chiefly to arithmetic are called arithmetic, portions devoted chiefly to algebra called algebra, and so on. This procedure would be fairly satisfactory for certain textbooks, but decidedly misleading for those in which the different parts of mathematics have been mixed to a considerable extent. In other examinations of textbooks, the procedure just mentioned has apparently been used most frequently, perhaps because it is the easiest method. A very good method would be to use the line or the vertical inch on the page as the unit, but such procedure would be very tedious and therefore not desirable for an examination of a large number of books. In the present investigation, the measurement was made in terms of the page. The exact procedure will be explained later in this chapter.

DIVISIONS OF THE SUBJECT MATTER IN THE ENTIRE STUDY

In general, the subject matter of the seventh, eighth, ninth, and tenth grades has been divided into five parts and some of the parts subdivided as follows:

1. *Arithmetic*: Wherever there is an apparent purpose to introduce arithmetical calculations or to connect the work in arithmetic with that of some other subject, the page is called arithmetic.
2. *Statistics*:
   (*a*) *Computational*—This includes the computational work in statistics and the theory of statistics, but not tables when the sole purpose is to give data for the construction of graphs.
   (*b*) *Graphs*—This includes all graphical work in which the data do not conform to some algebraic law which can be expressed in terms of an equation or formula.

3. *Algebra*:
   (*a*) *Numerical and Literal*—This includes the work which represents a distinct generalization of the concepts of arithmetic. Sometimes the distinction between arithmetic and numerical algebra has been rather difficult to draw.
   (*b*) *Graphs*—This includes all graphs of formulas, equations, and data which follow an algebraic law.
4. *Geometry*:
   (*a*) *Intuitive*—This includes all of the geometry in which there is no attempt to make formal proofs.
   (*b*) *Demonstrative*—This includes the geometry in which there are formal proofs.
5. *Trigonometry*: This includes all of the work in which there seems to be a definite purpose to present trigonometry, even though the subject is made up of arithmetic, algebra, and geometry.

If the entire page or all but five lines was devoted exclusively to one subject, it was counted as this subject entirely. It was thought best not to consider less than six lines of a subject because such a small amount would seem to vitiate the results more if included in the page survey than if omitted. If there were six or more lines of algebra, for example, on a page and the remainder intuitive geometry, the page was called algebra-intuitive geometry. If all but five lines or less of a page was intuitive geometry, the page was called intuitive geometry. Other subjects were treated in the same way. If more than five lines of each of three subjects was found on a page, all three subjects were listed in the page survey. The subject matter on many of the pages was mixed or fused. For instance, on one page may appear the rule for finding the area of a rectangle, the formula derived or stated, and examples to be solved by applying the formula. As long as one deals with the rectangle, he is certainly making use of intuitive geometry; the formula is algebra; and as soon as numbers are substituted in the formula the work becomes arithmetic. The writer asked several graduate students of mathematics what subject they would call a particular page in a ninth grade textbook. He received replies of "arithmetic," "algebra," and "intuitive geometry." Each student was ready to defend his decision. When the suggestion was made to call the page arithmetic-algebra-intuitive-geometry, they readily agreed and the page was so designated.

By the procedure just described, it was hoped to make the work almost entirely objective. It has been found, however, that in nearly every book new problems peculiar to that book have arisen which require decisions in the classification of certain pages. Hence a second study of the same textbooks would probably not reveal exactly the same count in number of pages devoted to the different subjects. The writer made an investigation of the reliability of results obtained from certain textbooks. He found that his results were fairly reliable, as will be shown later in this chapter. It seems reasonable to assume that the average of several books would be even more reliable than the results obtained from individual books.

PER CENTS OF DIFFERENT SUBJECTS

To obtain the number of pages of each subject in a textbook, it was necessary to divide in some way the pages on which two or more branches of mathematics were treated. As has already been explained, less than six lines of a subject on a page was not counted. When two subjects were attributed to a page, the page was divided equally between them. When three subjects were accredited to a page, each subject was given a credit of one-third of a page. The sum of the entire pages, half-pages, and one-third pages was then found. This gave the number of pages attributed to each subject. These numbers were divided by the total number of pages in the textbook, to get the per cent of the book devoted to each subject. Tables 11, 12, 13, and 14 give the per cents for the seventh and eighth grade textbooks. Such procedure does not give the exact per cents of each subject in a particular book, because in some cases more than one-half (or one-third) of a page is devoted to one subject and less to the other subject (or subjects). This method may give too much credit to one subject and too little to the other (or others). Usually the amounts exceeding that accredited to a subject will about balance that in which too little credit is given. This is more likely to be true when the averages of several books are considered than when the books are taken individually. Apparently no method can be devised which would be absolutely accurate, because not only are there varying portions of the page devoted to a subject, but the relative importance of the different subjects in the correlated or fused material varies just as greatly. Nevertheless, the results are sufficiently

reliable for comparison of individual textbooks, and probably quite reliable when the averages of five or more books are considered.

SEVENTH AND EIGHTH GRADE TEXTBOOKS

In the seventh and eighth grade textbooks, the only trace of computational statistics found concerned the arithmetic mean, commonly called the "average." Apparently it was thought of

TABLE 11

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN ARITHMETICS FOR THE SEVENTH AND EIGHTH GRADES PUBLISHED ABOUT 1900

Textbook No.	Arithmetic	Statistical Graph	Algebra			Intuitive Geometry	Trigonometry
			N. & L.	Graph	Total		
1	79.4	....	5.9	....	5.9	14.7	....
2	73.5	....	11.8	....	11.8	14.7	....
3	90.5	....	0.5	....	0.5	9.1	....
4	86.3	....	3.8	....	3.8	9.8	....
5	82.1	....	10.7	....	10.7	7.2	....
Mean ...	82.4	....	6.5	....	6.5	11.1	....

TABLE 12

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN ARITHMETICS FOR THE SEVENTH AND EIGHTH GRADES PUBLISHED ABOUT 1915

Textbook No.	Arithmetic	Statistical Graph	Algebra			Intuitive Geometry	Trigonometry
			N. & L.	Graph	Total		
6	86.3	....	6.0	....	6.0	7.7	....
7	84.3	....	6.7	....	6.7	9.0	....
8	79.2	....	3.8	....	3.8	17.0	....
9	79.4	3.3	6.1	....	6.1	11.1	....
10	77.9	0.8	10.1	....	10.1	11.0	....
11	87.3	1.9	3.5	....	3.5	7.4	....
Mean ...	82.4	1.0	6.0	....	6.0	10.5	....

TABLE 13

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN ARITHMETICS FOR THE SEVENTH AND EIGHTH GRADES PUBLISHED SINCE 1925

Textbook No.	Arithmetic	Statistical Graph	Algebra			Intuitive Geometry	Trigonometry
			N. & L.	Graph	Total		
12	84.4	4.5	1.8	....	1.8	9.3	....
13	82.3	2.9	4.9	....	4.9	9.8	....
14	75.9	5.6	5.1	....	5.1	13.4	....
15	89.3	8.5	2.1	....	2.1	....	....
16	75.5	3.8	9.7	....	9.7	11.0	....
17	76.2	3.2	7.9	....	7.9	12.7	....
18	73.2	2.8	9.5	....	9.5	14.5	....
19	86.9	2.4	2.9	....	2.9	7.7	....
20	93.6	...	...	....	...	6.4	....
21	72.3	5.0	7.7	....	7.7	14.9	....
22	74.7	6.7	8.5	....	8.5	10.2	....
Mean ...	80.4	4.1	5.5	....	5.5	10.0	....

TABLE 14

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN SEVENTH AND EIGHTH GRADE JUNIOR HIGH SCHOOL BOOKS PUBLISHED SINCE 1923

Textbook No.	Arithmetic	Statistical Graph	Algebra			Intuitive Geometry	Trigonometry
			N. & L.	Graph	Total		
23	62.1	3.8	14.6	1.4	16.0	17.7	0.5
24	73.1	4.2	4.9	2.1	7.0	15.7	...
25	58.0	5.0	9.9	1.9	11.8	23.9	1.4
26	50.9	3.5	9.3	0.6	9.9	35.8	...
27	63.5	6.8	12.8	0.6	13.4	16.3	...
28	51.8	3.2	21.5	...	21.5	23.6	...
29	63.2	1.8	18.5	0.2	18.7	16.4	...
30	32.1	3.8	29.4	2.4	31.8	31.4	0.8
31	67.6	6.0	8.9	0.2	9.1	17.2	...
32	66.0	5.1	12.5	1.1	13.6	15.3	...
Mean ...	58.8	4.3	14.2	1.1	15.3	21.3	0.3

as arithmetic rather than statistics. Hence it has not been given a separate place in the tables of this chapter, but has been included in the pages attributed to arithmetic. Demonstrative geometry was not found in any of the textbooks for classes below the ninth grade. For this reason, no place was assigned to it in these tables.

The seventh and eighth grade arithmetic textbooks contained algebra, statistical graphs, and intuitive geometry as well as arithmetic, as shown in Tables 11, 12, and 13. The per cent of each branch of mathematics in the junior high school textbooks investigated is given in Table 14.

RELIABILITY OF THE PER CENTS

The per cents in the tables are of no value unless they have a certain degree of reliability. The writer has made two separate investigations of all the junior high school textbooks. The first was completed about two months before the second was begun. In the first investigation, a subject was attributed to a page if any trace of it could be found thereon. In the second, the subject was ignored if less than six lines were found on a page. Even with this difference in procedure, there was not much variation in results. Out of the fifty-three different per cents obtained, none differed more than four per cent; only three, more than two per cent; and thirty-two, not more than one per cent. They would probably have differed less than here indicated if exactly the same standards had been used in both investigations. This indicates that the results are fairly reliable.

GENERAL MATHEMATICS IN SEVENTH AND EIGHTH GRADE TEXTBOOKS

1. *Seventh and Eighth Grade Mathematics in 1900.*—Table 11 shows the relative amounts of each of the different branches of mathematics at the beginning of the present century. In every one of the five books investigated, more than seventy per cent of the work was in arithmetic and the remainder was divided between algebra and intuitive geometry. There were no statistical or algebraic graphs and no trigonometry in any of them.

2. *Seventh and Eighth Grade Mathematics in 1915.*—There was very little change in the relative amounts of each branch of mathematics in these grades from 1900 to 1915. In three of the six

books published about 1915, there was a small amount of statistical graphs. There was no other change worth noting. Thus it appears that the movement for correlated general mathematics during this period of time did not extend down into the grades below the four-year high school.

3. *Seventh and Eighth Grade Mathematics in the Eighth-Year Elementary School Textbooks at the Present Time.*—A greater change was evident after 1915 than in the period just preceding. This was probably due to the junior high school movement and to the tendency to select some of the easier portions of high school mathematics for presentation to the grade children. The chief change was made in the inclusion of statistical graphs in all but one of these eleven textbooks of recent date. The amount was considerably greater than in the three textbooks of 1915 which contained graphs. A slight reduction was made in the amount of each of the other three branches to provide space for this new subject. There were greater differences of opinion among the authors of recent date than in both of the preceding periods as to how much of each subject should be included in the textbook. One author went so far as to omit all traces of algebra. Another included no work in intuitive geometry. None of the books for any of the three periods contained any algebraic graphs or numerical trigonometry.

4. *Mathematics in the Textbooks for the First Two Years of the Junior High School.*—The junior high school textbooks for the grades now being considered contained about the same amount of statistical graphs as the books written for the eight-year elementary schools. In every other branch, there was a very noticeable difference. Arithmetic dropped from an average of about 80 per cent in the arithmetics for elementary schools to about 59 per cent in the junior high school textbooks. Algebra increased from 5.5 per cent to 15.3 per cent and intuitive geometry from 10 per cent to 21.3 per cent. There was a trace of algebraic graphs in all the junior high school textbooks investigated except one and also a little numerical trigonometry in three of the ten books.

HOW THE CORRELATIONS WERE OBTAINED

The original tabulations of the pages to which two or three branches were attributed were used in computing Tables 15, 16, 17, and 18. These tables give the amount of correlation or mixing

of the branches in the different books and for different periods of time. The method to be described here does not distinguish between correlating the subjects and mixing them. The writer believes that to do so would be very difficult. The results obtained, however, show very definitely the attempts of the authors to treat together the subject matter of the different branches of mathematics. In the discussions in the present chapter and the next two chapters, such procedure will be referred to as correlation.

TABLE 15

PER CENT OF TOTAL PAGES CORRELATED AND THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN THE SEVENTH AND EIGHTH GRADE TEXTBOOKS PUBLISHED ABOUT 1900

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Miscellaneous
1	27.6	5.1	22.8	4.6	....
2	9.4	2.7	7.9	1.2	....
3	16.8	1.0	16.2	0.3	....
4	15.6	1.5	15.3	1.2	....
5	19.2	7.7	12.5	3.6	....
Mean	17.7	3.6	14.9	2.2	....

TABLE 16

PER CENT OF TOTAL PAGES CORRELATED AND THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN THE SEVENTH AND EIGHTH GRADE TEXTBOOKS PUBLISHED ABOUT 1915

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Miscellaneous
6	14.4	5.1	10.9	4.4	....
7	14.0	2.8	11.9	1.3	....
8	33.9	9.8	30.8	6.7	....
9	25.4	8.4	19.1	2.3	0.7
10	20.0	2.0	17.5	0.6	....
11	16.8	3.5	13.8	0.5	....
Mean	20.8	5.3	17.3	2.6	0.1

TABLE 17

PER CENT OF TOTAL PAGES CORRELATED AND THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN THE SEVENTH AND EIGHTH GRADE TEXTBOOKS PUBLISHED SINCE 1925

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Miscellaneous
12	14.2	1.4	12.6	0.9	0.7
13	10.0	2.4	8.2	2.4	....
14	20.3	4.0	11.0	7.5	4.3
15	15.8	4.3	....	....	11.5
16	15.4	6.6	10.9	4.8	....
17	20.5	4.1	17.9	2.6	0.2
18	16.8	7.8	14.1	7.0	0.3
19	15.3	4.5	11.0	2.5	2.2
20	9.5	....	9.5	....	....
21	21.1	9.3	17.5	9.8	2.6
22	25.9	14.3	14.1	9.2	5.4
Mean	16.8	5.3	11.5	4.2	2.5

TABLE 18

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS ARE CORRELATED IN THE JUNIOR HIGH SCHOOL TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Miscellaneous
23	23.7	11.8	16.4	9.1	1.2
24	21.1	3.5	13.8	7.0	2.8
25	19.0	9.2	10.8	11.0	1.8
26	27.0	13.6	17.5	19.1	1.4
27	23.6	15.9	17.0	15.5	1.5
28	19.9	8.8	10.7	13.9	....
29	20.3	10.8	12.1	8.2	1.0
30	31.1	14.4	14.6	22.8	2.2
31	33.8	17.9	21.3	17.4	6.6
32	19.9	13.5	16.7	11.8	0.4
Mean	23.9	11.9	15.1	13.6	1.9

The total per cent of correlation in a textbook was found by ascertaining the number of pages on which two or three branches had been listed and then dividing this by the total number of pages in the book. To find the proportion of the book in which two particular subjects such as arithmetic and geometry were correlated, the number of pages attributed to arithmetic and geometry and also to arithmetic, geometry, and some other subject was found and this number was divided by the total number of pages.

By this procedure, certain pages were attributed to two subjects when the subject matter was distinctly separated. It sometimes happened that one part of a subject was completed and the author introduced a new subject on the same page with no intention of correlating or combining them. In such case, this method of procedure would make the correlation seem too great. This would be more noticeable when the per cent of correlation was quite small. On the other hand, no provision was made to include those pages on which there were five lines or less of a subject. Hence it is probable that the amount of correlation as computed, except for very small per cents, is about right.

CORRELATION IN THE SEVENTH AND EIGHTH GRADE TEXTBOOKS

1. *Correlation of Subjects about 1900.*—In the textbooks published about 1900, an average of approximately one-sixth of the pages showed a correlation or mixing of two or more subjects. All of this was among the subjects of arithmetic, algebra, and geometry. The greatest amount of correlation was between arithmetic and geometry, about one-seventh of the pages containing both of these subjects.

2. *Correlation of Subjects about 1915.*—There was practically no change in the amount of work given to the different subjects from 1900 to 1915. There was, however, an increase in the relative number of pages which contained two or more subjects. The number increased from 17.7 per cent of the pages to 20.8 per cent. The greatest increase was in the correlation of arithmetic and geometry. The correlation of these two subjects increased from 14.9 per cent of the pages to 17. 3 per cent. There was a tendency, nevertheless, to increase the amount of correlation among all of the subjects.

3. *Correlation of Subjects at the Present Time.*—The seventh and eighth grade textbooks of to-day do not contain so much

correlation of subjects as those published in 1915. It is even slightly less than in the textbooks of 1900. In the eleven textbooks of recent date examined, the number of pages showing correlation of subjects was 16.8 per cent, in the six books of about 1915 it was 20.8 per cent, and in the five books of about 1900 it was 17.7 per cent. This recent decrease was not due to the mathematics being confined more particularly to one subject and thus making correlation impossible. In fact, there was a slight increase in the variety of subject matter and hence in the general nature of the mathematics work. The decrease was probably due to a tendency away from so much correlation of mathematical subjects.

The junior high school textbooks of to-day for the seventh and eighth grades show 23.9 per cent of the pages correlated. This amount is greater than the amount correlated in the books for the eight-year elementary schools. The textbooks for the junior high schools contain a much greater variety of subject matter than those for the eight-year elementary schools, thereby making more correlation possible. When this fact is taken into consideration, the relative amount of correlation in the textbooks for these two different types of schools is about the same.

TRENDS IN THE SEVENTH AND EIGHTH GRADE TEXTBOOKS

The contents of the textbooks examined indicate certain trends in seventh and eighth grade mathematics. These will be discussed under three heads, as follows:

1. *Trend toward the Junior High School Organization.*—No doubt the junior high school movement will have considerable influence, since an ever-increasing number of schools are organized on the 6–3–3 plan. It is probable, therefore, that there will be a corresponding increase in the number of junior high school textbooks and that these will have a greater influence as the number of junior high schools increases. Most of the junior high school textbooks are excellent examples of general mathematics. An increase in the number of junior high schools would consequently mean that more schools would offer general mathematics in the seventh and eighth grades.

2. *Trend toward a More General Mathematics in the Eight-Year Elementary School.*—It seems obvious that the junior high school subject matter must exert some influence upon the material included in the textbooks for the eight-year elementary schools.

Already there is some tendency to make the seventh and eighth grade textbooks approach a type of general mathematics. A few series of textbooks such as No. 21 and No. 22 in this investigation are almost as good types of general mathematics as the junior high school textbooks. This indicates that certain authors have a vision of giving a course in mathematics to the pupils in schools organized on the 8–4 plan similar to that which is being offered to the junior high school pupils. Whether or not the mathematics for these two types of schools will ever become the same is uncertain. It is quite likely, however, that there will be less difference in the future than at present and that the junior high school will exert the greater influence in bringing about this change.

3. *Trend away from Emphasizing Correlation.*—The correlation of mathematical subjects shows a tendency of becoming firmly established. However, there is somewhat less correlation now than there was fifteen years ago. Correlation was so much emphasized then that many correlations have been forced. Authors are now concerned chiefly in presenting the most valuable subject matter of mathematics for the grades in question. For the most part they try to relate the subjects only where the connections can be made easily and naturally.

CHAPTER VI

A CRITICAL STUDY OF NINTH GRADE TEXTBOOKS

The study made of ninth grade mathematics textbooks was very similar to that of seventh and eighth grade textbooks. The methods of procedure were identical. Investigations were made of textbooks written about 1900, about 1915, and at the present time. The work in algebra which has become the core of ninth grade mathematics took definite form in the eighteenth and nineteenth centuries. The writer was interested to know how the books of these two centuries differed from those at the beginning of the present century. For this reason and because the present investigation is primarily concerned with ninth and tenth grade mathematics, an examination was made of two textbooks published in the eighteenth century and two in the nineteenth century.

NINTH GRADE TEXTBOOKS EXAMINED

In order to investigate the problems set forth in this and the preceding chapter with respect to ninth grade mathematics, the writer made a critical examination of the following textbooks: (1) a widely used college mathematics textbook, and also one of the earliest algebras published; (2) two of the best-known algebra books of the nineteenth century; (3) five algebra books published between 1896 and 1902 inclusive; (4) seven algebra books published between 1913 and 1917 inclusive; (5) twelve algebra books published since 1923; (6) eight of the most important ninth grade general mathematics textbooks of the present century; and (7) ten ninth grade junior high school textbooks published since 1923. This makes a total of forty-six books upon which the report of this chapter is based. In each of the periods since 1896, all but one of the books in the Library of Teachers College, Columbia University, in the summer of 1928 were examined. The exception was a junior high school textbook which was nearly all arithmetic. The reason for omitting it has been explained on page 54.

GENERAL MATHEMATICS IN NINTH GRADE TEXTBOOKS

1. *General Mathematics Prior to the Twentieth Century.*—One of the best examples of general mathematics is Ward's *Mathematics*, which was used extensively in the American colleges of the eighteenth century. The book consisted of five parts. The present investigation has been confined to the first three parts; namely, arithmetic, algebra, and geometry. Conic sections and the arithmetic of infinites, the subject matter of the third and fourth parts, were not investigated in this study because they are now generally

TABLE 19

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN FOUR TEXTBOOKS OF THE EIGHTEENTH AND NINETEENTH CENTURY

Textbook No.*	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
33 ...	43.7	.....	.....	.....	39.1	.....	39.1	0.6	16.6	17.2	.....
34 ...	0.5	.....	.....	.....	92.2	.....	92.2	.....	7.3	7.3	.....
35 ...	1.3	.....	.....	.....	97.9	.....	97.9	0.8	.....	0.8	.....
36 ...	4.3	.....	.....	.....	95.7	.....	95.7	.....	.....	.....	.....

* The four textbooks indicated by number in this table are as follows:

33. John Ward, *The Young Mathematician's Guide; Being a Plain and Easy Introduction to the Mathematicks* (Sixth Edition, 1723).

34. Nathaniel Hammond, *The Elements of Algebra in a New and Easy Method* (1772).

35. Warren Colburn, *An Introduction to Algebra* (1834).

36. Charles Davies, *Elementary Algebra* (1852).

outside the realm of ninth grade mathematics. A separate course in algebra began to be offered in college in the latter part of the eighteenth century. Textbook number 34 was chosen as a representative algebra of this period. Table 19 shows that about 92 per cent of this book was algebra and most of the remainder was geometry. This was nearly two and one-half times as much algebra as was contained in Ward's *Mathematics* of less than half a century earlier. The two books of the nineteenth century, numbers 35 and 36, consisted almost wholly of algebra, and represented the period in which the compartment idea of mathematics for instruction in the secondary schools of America was almost universally accepted. A few books did include a unit of numerical

trigonometry, but these were not investigated in the present study because trigonometry was rarely taught in the ninth grade. Most of the books containing trigonometry appeared about the middle of the nineteenth century.

2. *General Mathematics at the Beginning of the Present Century.*—Table 20 reveals that the same conception of ninth grade mathematics was held by textbook writers at the beginning of the present

TABLE 20

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN NINTH GRADE ALGEBRA TEXTBOOKS PUBLISHED ABOUT 1900

Textbook No.	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
37	3.8	.....	.....	.....	95.5	.....	95.5	0.7	.....	0.7	.....
38	2.0	.....	.....	.....	97.7	.....	97.7	0.3	.....	0.3	.....
39	1.8	.....	.....	.....	96.3	.....	96.3	1.9	.....	1.9	.....
40	1.3	.....	.....	.....	95.1	3.6	98.7	.....	.....	.....	.....
41	2.3	.....	.....	.....	97.7	.....	97.7	.....	.....	.....	.....
Mean	2.2	.....	.....	.....	96.5	0.7	97.2	0.6	.....	0.6	.....

century as was held during the nineteenth century. At least 95 per cent of the subject matter of each of the five textbooks examined was algebra. Neither trigonometry nor statistics was found in any one of them.

3. *General Mathematics Textbooks of the Present Century.*—The general mathematics textbooks (See Table 23) had an average of only about 64 per cent algebra, 24 per cent geometry and a little arithmetic, statistics, and numerical trigonometry. The two books published prior to 1916 (numbers 61 and 62) did not contain any trigonometry. Since that time every general mathematics textbook has contained a unit of this subject. The subject of statistical graphs was found in every one of these books, but computational statistics was not contained in any of those published before 1918. Books number 64, 65, 66, and 67 have usually been called general mathematics and the others correlated mathematics. There is, however, no very clear distinction between the

two types. As far as the per cent devoted to the different subjects in mathematics is concerned, there was no real difference. Some distinction will be shown later when correlation is discussed.

4. *Effect of the Correlated Mathematics Movement on Algebra Textbooks.*—For the effect of the correlated mathematics movement on algebra textbooks, the per cents recorded in Tables 20 and 21 may be compared. Of the textbooks examined, the proportion of books devoted to algebra dropped between 1900 and 1915 from 97.2 per cent to 89.3 per cent, the proportion devoted to

TABLE 21

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN NINTH GRADE ALGEBRA TEXTBOOKS PUBLISHED ABOUT 1915

Textbook No.	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
42 ...	6.1	.....	0.7	0.7	81.7	5.2	86.9	6.3	.....	6.3	.....
43 ...	4.0	.....	1.1	1.1	84.0	4.7	88.7	6.2	.....	6.2	.....
44 ...	4.8	.....	0.6	0.6	85.9	2.4	88.3	6.3	.....	6.3	.....
45 ...	5.6	.....	2.8	2.8	80.4	4.5	84.9	6.6	.....	6.6	.....
46 ...	6.3	.....	2.5	2.5	77.2	10.1	87.3	3.9	.....	3.9	.....
47 ...	4.1	.....	1.8	1.8	88.7	3.5	92.2	1.8	.....	1.8	.....
48 ...	1.8	.....	.....	.....	95.2	1.4	96.6	1.6	.....	1.6	.....
Mean	4.7	.....	1.4	1.4	84.7	4.5	89.3	4.7	.....	4.7	.....

arithmetic increased from 2.2 per cent to 4.7 per cent, and the proportion devoted to geometry increased from 0.6 per cent to 4.7 per cent. Statistical graphs was beginning to be introduced in 1915. Numerical trigonometry had not yet appeared in the algebra textbooks. From this examination, it is apparent that the agitation for correlated mathematics was having some effect upon the traditional algebra course, and that a small portion of the algebra was being replaced by arithmetic, statistical graphs, and intuitive geometry.

5. *General Mathematics in the Junior High School Textbooks.*—The per cent of algebra in the recent junior high school textbooks, as revealed by Table 24, was 70.4, which was only about 7 per cent greater than in the general mathematics books. The seventh and

TABLE 22

PER CENT OF DIFFERENT BRANCHES OF MATHEMATICS IN NINTH GRADE TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
49 ...	6.0	.....	3.9	3.9	69.9	6.0	75.9	8.7	.....	8.7	5.5
50 ...	4.2	.....	2.4	2.4	82.1	5.4	87.5	6.0	.....	6.0	.....
51 ...	4.8	.....	3.0	3.0	80.8	2.8	83.6	5.2	.....	5.2	3.4
52 ...	5.4	0.5	3.0	3.5	74.9	4.3	79.2	7.3	.....	7.3	4.6
53 ...	5.9	0.3	5.1	5.4	70.7	5.3	76.0	7.7	.....	7.7	5.0
54 ...	2.0	.....	5.8	5.8	79.5	3.2	82.7	5.9	.....	5.9	3.5
55 ...	6.3	.....	5.0	5.0	72.4	3.3	75.7	6.4	.....	6.4	6.6
56 ...	5.3	.....	3.3	3.3	76.5	4.7	81.2	6.8	.....	6.8	3.3
57 ...	5.9	1.7	5.2	6.9	70.7	4.2	74.9	6.9	.....	6.9	5.4
58 ...	4.9	.....	2.4	2.4	80.7	3.2	83.9	4.7	.....	4.7	4.1
59 ...	5.6	1.4	3.7	5.1	74.0	7.2	81.2	4.8	.....	4.8	3.3
60 ...	3.8	.....	3.5	3.5	80.3	5.9	86.2	4.0	.....	4.0	2.5
Mean	5.0	0.3	3.9	4.2	76.0	4.6	80.7	6.2	.....	6.2	3.9

TABLE 23

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN EIGHT NINTH GRADE GENERAL MATHEMATICS TEXTBOOKS OF THE PRESENT CENTURY

Textbook No.	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
61 ...	6.3	.....	1.7	1.7	60.2	4.0	64.2	22.0	5.9	27.9	.....
62 ...	7.5	.....	4.6	4.6	59.5	9.0	68.5	19.2	0.2	19.4	.....
63 ...	4.4	.....	1.9	1.9	49.6	2.9	52.5	33.5	6.5	40.0	1.2
64 ...	7.6	.....	1.7	1.7	67.1	4.6	71.7	17.9	.....	17.9	1.1
65 ...	3.3	3.6	5.6	9.2	58.5	8.1	66.6	14.7	.....	14.7	6.2
66 ...	3.4	4.9	5.4	10.3	49.5	6.8	56.3	25.7	2.0	27.7	2.3
67 ...	4.8	0.4	2.7	3.1	62.8	4.0	66.8	22.1	.....	22.1	3.2
68 ...	4.8	.....	5.3	5.3	58.6	3.2	61.8	25.1	.....	25.1	3.0
Mean	5.3	1.1	3.6	4.7	58.2	5.3	63.5	22.5	1.8	24.4	2.1

eighth grade junior high school textbooks contain considerable algebra. Hence this reduction in the amount contained in the ninth grade books does not mean that the pupils' knowledge of the subject will be any less than under the traditional arrangement. As much work is given in the subject and it is distributed over a

TABLE 24

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN NINTH GRADE JUNIOR HIGH SCHOOL MATHEMATICS TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.*	Arithmetic	Statistics			Algebra			Geometry			Trigonometry
		Computational	Graph	Total	N. & L.	Graph	Total	Intuitive	Demonstrative	Total	
69 ...	8.3	.....	3.3	3.3	73.1	4.3	77.4	6.5	.....	6.5	4.6
70 ...	10.3	3.5	1.5	5.0	62.5	6.0	68.5	11.2	.....	11.2	5.1
71 ...	3.0	.....	.....	.....	56.2	9.9	66.1	15.6	7.7	23.3	7.7
72 ...	10.5	.....	.....	.....	61.2	5.4	66.6	16.7	2.8	19.5	3.5
73 ...	11.4	.....	1.5	1.5	61.3	9.0	70.3	9.9	.....	9.9	6.9
74 ...	13.6	.....	.....	.....	55.3	3.1	58.4	4.1	17.1	21.2	6.9
75 ...	6.0	1.8	5.2	7.0	71.6	6.6	78.2	5.0	.....	5.0	3.8
76 ...	3.5	.....	.....	.....	79.5	8.7	88.2	2.6	.....	2.6	5.7
77 ...	14.8	1.1	1.8	2.9	57.3	3.0	60.3	12.8	3.5	16.3	5.7
78 ...	5.2	1.3	3.7	5.0	66.5	3.4	69.9	8.7	6.1	14.8	5.1
Mean	8.7	0.8	1.7	2.5	64.5	5.9	70.4	9.3	3.7	13.0	5.5

* These are the same series of books and are arranged in the same order as those in Table 14 on page 59.

longer period of time. Somewhat more arithmetic has been introduced than in the general mathematics books, less statistical graphs, not quite so much intuitive geometry, and more numerical trigonometry. The seventh and eighth grade junior high school textbooks have more statistical graphs and more intuitive geometry than the other type of textbooks. Hence it was to be expected that not so much attention would be paid to these subjects in the ninth grade junior high school textbooks. Computational statistics and demonstrative geometry have not yet been assured places in either the general mathematics or junior high school textbooks. Some authors have included a unit of one or both of these subjects and others have omitted them. There is not any agreement at present as to whether they should be omitted or not.

6. *Effect of General Mathematics and Other Causes on Algebra Textbooks.*—The causes of the changes made in the algebra textbooks from 1915 to the present time are much more complicated than of the period before 1915. Three of the most direct causes are the recent movement for general mathematics, the National Committee's report, and the junior high school movement. These and other causes have been discussed more fully in Chapter III. Tables 21 and 22 show the effects of these causes upon the algebra textbooks. The proportion devoted to algebra decreased from 89.3 per cent in 1915 to 80.7 per cent in 1928. This is still about 10 per cent more than in the junior high school books and about 17 per cent more than in the regular general mathematics books. During this period the amount of arithmetic did not change much, statistical graphs almost trebled, intuitive geometry increased a little, and numerical trigonometry gained a sure place. A little computational statistics was introduced into a few books, and demonstrative geometry was consistently refused a place in all.

CORRELATION OF MATHEMATICS IN NINTH GRADE TEXTBOOKS

The per cents of pages correlated were found in the manner described in Chapter V. The textbooks were given the same number in the tables giving data on correlation (Tables 25, 26, 27, 28, 29, and 30) as in the tables giving data on the different branches of mathematics (Tables 19, 20, 21, 22, 23, and 24). Some of the most important facts revealed by the tables giving the per cents of correlation will now be discussed.

1. *Correlation of Subjects Prior to the Present Century.*—In Ward's *Mathematics*, the eighteenth century college textbook already considered, the subject matter on 24 per cent of the pages was correlated. (See Table 25.) This is less than the per cent in the so-called correlated mathematics books of to-day and more than in the so-called general mathematics books. This correlation was confined to arithmetic and algebra and to algebra and geometry. The eighteenth century algebra textbook (number 34) investigated also showed considerable correlation (about 15 per cent), nearly all of which was between algebra and geometry. The two nineteenth century textbooks (numbers 35 and 36) showed very little correlation. There could not be much correlation when nearly all of the material was algebra, as was the case in those two textbooks.

2. *Correlation of Subjects in the General Mathematics Textbooks of the Present Century.*—The different subjects were found to be correlated in the general mathematics books of the present century, as shown by Table 29, on slightly more than 35 per cent of the pages for each one of the first four books listed and 31 per cent for the last book in this group. All five of these have been called correlated mathematics except number 64, which has often been designated as a general mathematics textbook. Book number 64 really belongs to the group of correlated mathematics books. The remaining three textbooks (numbers 65, 66, and 67) showed correlation of subjects on about 20 per cent of the pages. This was decidedly less than in the other books of the group. These three books represented a modification of the extreme type of correlation and, at the same time, retained as great a variety of material as correlated mathematics offered. Textbook number 68 was a recently published textbook written by the same author as number 62, published in 1916, and showed a slight reduction in the amount of correlation. This is quite natural in view of the recent developments of mathematics. The entire group of eight books showed an average correlation of algebra and geometry on about 25 per cent of the pages, of arithmetic and algebra on about

TABLE 25

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN FOUR TEXTBOOKS OF THE EIGHTEENTH AND NINETEENTH CENTURIES

Textbook No.*	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
33	24.0	12.6	....	....	11.5	....
34	15.5	0.9	....	....	14.6	....
35	3.2	1.6	....	....	1.6	....
36	3.5	3.5	....	....	....	....

* These textbooks are the same as those reported in Table 19 on page 68.

8 per cent, and of arithmetic and geometry on about 4 per cent. The correlation of other subjects was very small or of such a nature that the present investigation would not show the amount.

A correlation of arithmetic and statistics is not to be expected from these tables because most of the elementary statistics suitable

for the ninth grade is really a special part of arithmetic. Since numerical trigonometry is made up of arithmetic, algebra, and geometry, it should not be expected that much correlation would be revealed between it and any one of the other three subjects. The correlation as shown of statistics and algebra was chiefly due to the relationship between statistical and algebraic graphs. The same statement can be made concerning the relationship of these two subjects as shown in the other tables.

TABLE 26

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN NINTH GRADE ALGEBRA TEXTBOOKS PUBLISHED ABOUT 1900

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
37	5.2	3.8	...	...	1.3	...
38	3.5	2.9	...	...	0.6	...
39	6.1	2.7	0.5	...	4.0	...
40	1.9	1.9	...	...	...	...
41	3.8	3.8	...	...	...	...
Mean	4.1	3.0	0.1	...	1.2	...

TABLE 27

PER CENT OF TOTAL PACES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN NINTH GRADE ALGEBRA TEXTBOOKS PUBLISHED ABOUT 1915

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
42	22.0	12.2	5.1	...	13.5	...
43	18.6	5.6	1.1	0.7	13.3	...
44	19.9	8.5	2.0	...	13.4	...
45	24.1	10.2	2.6	1.9	13.9	...
46	18.3	12.9	3.5	...	8.9	...
47	9.5	6.0	...	...	3.6	...
48	5.9	2.6	...	...	3.3	...
Mean	16.9	8.3	2.0	0.4	7.0	...

TABLE 28

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN NINTH GRADE ALGEBRA TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
49	22.1	10.1	5.2	...	17.2	...
50	19.0	8.7	4.2	0.6	13.3	...
51	15.5	7.6	2.0	...	9.9	0.6
52	21.5	11.1	2.8	0.5	12.4	0.3
53	22.0	8.0	1.9	0.6	13.7	1.5
54	17.4	4.6	1.4	2.0	12.6	0.6
55	19.0	9.5	3.7	0.3	11.9	0.3
56	16.3	6.6	4.2	...	12.1	...
57	18.5	9.6	4.4	...	12.8	0.5
58	17.5	8.9	1.4	...	10.0	...
59	18.4	8.5	2.5	1.1	10.5	0.9
60	12.0	6.1	2.3	0.7	7.4	...
Mean	18.3	8.3	3.0	0.5	12.0	0.4

TABLE 29

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN EIGHT NINTH GRADE GENERAL MATHEMATICS TEXTBOOKS OF THE PRESENT CENTURY

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
61	35.3	12.3	5.5	0.8	26.3	...
62	36.7	10.2	2.5	...	29.0	...
63	36.0	9.5	6.3	0.6	30.1	1.8
64	37.0	5.9	6.3	1.7	34.3	0.4
65	19.4	5.5	1.6	0.5	14.5	0.6
66	23.1	6.0	3.5	0.2	18.8	0.4
67	22.0	8.4	5.8	0.4	17.6	0.4
68	31.0	7.1	2.8	1.5	24.5	1.2
Mean	30.1	8.1	4.3	0.7	24.4	0.6

3. *Changes in Amount of Correlation of Subjects in Algebra Textbooks from 1900 to 1915.*—The correlated mathematics movement, as shown by Tables 26 and 27, had a decided effect upon the correlation of subjects in textbooks published about 1915. The proportion of pages on which mathematical subjects were correlated increased between 1900 and 1915 from 4.1 per cent to about 16.9 per cent. In other words it was approximately four times as great in 1915 as in 1900. The increase was chiefly in the correlation of arithmetic and algebra and of algebra and geometry. The correlations of all other subjects were small.

4. *Correlation of Subjects in Junior High School Textbooks.*—The correlation of subjects in the junior high school textbooks was 23.5 per cent of the pages. (See Table 30.) It was 6.6 per cent less than the average of the general (including correlated) mathematics books. This was due chiefly to less variety of subject matter in the junior high school textbooks. This shows that the authors of this type of textbooks have adopted very largely not only the principle of general mathematics but also that of correlation whenever they can use it. The difference in correlation of subjects between the two groups of books was due to less correlation of algebra and geometry in the junior high school books. The

TABLE 30

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH DIFFERENT SUBJECTS ARE CORRELATED IN THE NINTH GRADE JUNIOR HIGH SCHOOL TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Statistics-Algebra	Algebra-Geometry	Miscellaneous
69	20.9	9.9	1.7	0.7	11.6	0.9
70	30.1	13.3	5.9	0.5	19.5	1.1
71	30.7	5.4	0.4	...	25.6	0.8
72	23.6	9.7	6.3	...	18.8	...
73	34.0	17.2	2.6	1.0	17.8	1.2
74	12.0	5.6	1.9	...	8.0	...
75	20.6	8.1	0.6	2.9	10.0	1.5
76	10.6	4.8	1.3	...	4.8	...
77	33.4	14.7	6.5	1.4	22.3	1.0
78	19.0	9.6	4.1	...	13.2	0.3
Mean	23.5	9.8	3.1	0.7	15.2	0.7

amount of correlation between other subjects did not differ very much in the two groups of books.

5. *Correlation of Subjects in the Present Algebra Textbooks.*— The tables (Tables 27, 28, 29, and 30) show that the amount of correlation of subjects in the recent algebra books was about three-fifths as great as in the general mathematics books, about 5 per cent less than in the junior high school books, and only slightly more than in the algebras of 1915. In the recent algebra books it totaled 18.3 per cent. The per cent of correlation did not increase as fast as the tendency toward general mathematics made possible. This can be seen by an examination of the tables. There has been a considerable increase, however, in the correlation of algebra and geometry but not much change in that of other subjects.

TRENDS IN NINTH GRADE TEXTBOOKS

In the light of this investigation of textbooks, certain statements can be made about what seem to be the present trends of ninth grade mathematics. The writer is aware that new factors which will entirely change these trends may enter at any time.

1. There is a decided tendency among authors of the ninth grade algebras to introduce more material from the different branches of mathematics into the course, but not to increase the relative amount of correlation of subjects. Algebra, however, will probably continue to be the chief subject for study in the ninth grade.

2. The tendency in general mathematics books is not to correlate the subjects as much as was done a few years ago, but the rich variety of material will probably be retained. The junior high school movement is too new to indicate the trend in junior high school mathematics; it will probably go in the same direction as general mathematics, but with the work organized on the three-year course as a unit.

3. More intuitive geometry will probably be introduced into the regular algebras than is included at present. This will probably be used to strengthen the course in algebra. It is valuable in the study of algebraic formulas and in problem work. The amount of geometry in the algebra textbooks is far less than in either general mathematics books or junior high school books.

4. No marked change is indicated in the present amount of arithmetic in the ninth grade. Whatever is taught in the future

will probably be in direct connection with some other subject or in work which is preparatory to statistics, rather than as an independent unit of work.

5. The future position of computational statistics is uncertain. There is a tendency among certain authors to include some work in this subject. It may be that a greater number of authors will include this subject in future books, not so much because of the precedent as because of the increasing interest in the subject in the educational field.

6. The work in statistical graphs is more likely to decrease than to increase. This trend is revealed by the junior high school textbooks which contain less of this material than either the general mathematics books or the recent algebras. This is apparently due to the fact that more graphical work is done in the seventh and eighth grades in schools where junior high school textbooks are used than in schools where the regular arithmetic books are used. As soon as the majority of the arithmetic books take over the work in statistical graphs, the amount of this subject in the ninth grade will probably be diminished.

7. Algebraic graphs will probably retain its present position in the ninth grade. The seventh and eighth grade textbooks are leaving this subject for the ninth grade. This tendency in regard to algebraic graphs is about the same in each of the recently published groups of books. This does not mean, however, that there will not be a change in the nature of the work in algebraic graphs.

8. Some tendency is in evidence to include a unit of demonstrative geometry in the ninth grade mathematics. This has been confined entirely to the general mathematics and junior high school textbooks. It is too early to predict whether or not this subject will gain a firm foothold in the first year of the regular four-year high school. The strength of the movement has been due chiefly to the fact that the pupils have already had a sufficient background in intuitive geometry to warrant, in the opinion of certain writers, the introduction of demonstrative geometry toward the end of this grade, and to the desire to give the ninth grade pupils an introduction to this new branch of mathematics. Recently the New York Regents have included some work in this subject in the syllabus for the ninth grade of the junior high school. This may have considerable influence in the future dis-

position of the introduction of demonstrative geometry. This syllabus will be discussed in Chapter VIII.

9. Numerical trigonometry has secured a sure place in the ninth grade. Its position, according to present indications, is not likely to change much in the immediate future.

CHAPTER VII

A CRITICAL STUDY OF TENTH GRADE TEXTBOOKS

A study of tenth grade textbooks similar to those recorded in Chapters V and VI will be reported in this chapter. The method of procedure and the problems for investigation have been described in Chapter V. In addition to textbooks of the three periods of time mentioned there, two geometry books of the nineteenth century were examined by the writer.

TEXTBOOKS EXAMINED

A critical examination was made of the following tenth grade textbooks: (1) two plane geometry books published in the nineteenth century; (2) five plane geometry books published between 1899 and 1901 inclusive; (3) five plane geometry books published between 1913 and 1916 inclusive; (4) ten plane geometry books published since 1923; and (5) six general mathematics books for the tenth grade published in the twentieth century. These books include all those which were available in the Teachers College Library of Columbia University in the summer of 1928 for the different periods of time. The present chapter will include general conclusions of a study of all these textbooks. A total of one hundred six have been examined, if the seventh and eighth grade books are counted as one book even though they are published separately. Thirty-two of these were seventh and eighth grade books; forty-six, ninth grade books; and twenty-eight, tenth grade books.

RELIABILITY OF THE PER CENTS OF THE BRANCHES

An investigation was made to determine the reliability of the results in the study of seventh and eighth grade textbooks. Two separate examinations were made of all the junior high school textbooks. A report of this is given in Chapter V. The writer found the textbooks of tenth grade general mathematics the most difficult of all to classify. Four of these were selected for repeated

examination. The results obtained did not differ in any instance by more than approximately 4 per cent. The per cents of branches differed in three cases by about 4, in four cases by about 2, in ten cases by about 1, and in fourteen cases by not more than .5. This does not include the zero per cents. These results show that the per cents were fairly reliable for individual textbooks and apparently much more reliable when the average of several books was found.

GENERAL MATHEMATICS IN THE TENTH GRADE TEXTBOOKS

The subject matter of mathematics naturally changes from grade to grade. This created new difficulties in the investigation of tenth grade textbooks. Statistics was found in only one of them, and amounted to only three-tenths of one per cent. This small trace of statistics was statistical graphs and was placed in the tables with algebraic graphs. Statistics as such was not given a place in the tables.

A separation of intuitive and demonstrative geometry was attempted, but proved to be almost impossible. In certain cases the geometry could not be classed as either kind, and in other cases the distinction was questionable. This classification was finally abandoned and a separation into plane and solid geometry took its place.

Another frequent difficulty was in regard to what kinds of mathematics were contained on certain pages of problem work. The ultimate decision was as follows: first, if the geometric problems could apparently be more easily solved by arithmetic, a page containing such problems would be called arithmetic-plane (or solid) geometry; second, if arithmetic and algebra would both be used in their solutions, the page would be called arithmetic-algebra-plane (or solid) geometry; and third, if algebra would be used chiefly in their solutions, the page would be called algebra-plane (or solid) geometry. Even this plan involved some rather difficult decisions.

1. *Extent of General Mathematics in the Plane Geometry Textbooks at the Beginning of and Prior to the Present Century.*—Table 32 shows that approximately 90 per cent of the work in the tenth grade geometry textbooks published at the beginning of the present century was plane geometry. Only a small per cent of the material was solid geometry. This consisted not of demonstrative solid

geometry, but of definitions placed chiefly in the introduction to the course. Usually no further use was made of these definitions. Of the 10.6 per cent of the work which was not geometry, the major portion (8.8 per cent) was algebra. Most of this was the algebra of proportion. The algebraic treatment of proportion is

TABLE 31

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN TWO PLANE GEOMETRY TEXTBOOKS PUBLISHED IN THE NINETEENTH CENTURY

Textbook No.*	Arithmetic	Algebra			Geometry			Trigonometry
		N. & L.	Graph	Total	Plane	Solid	Total	
79	...	2.3	...	2.3	97.7	...	97.7	...
80	2.6	11.9	...	11.9	85.4	...	85.4	...

* These two textbooks were: 79, Benj. Greenleaf, *New Elementary Geometry* (1873) and 80, J. B. Thompson, *Elements of Geometry* (1844).

TABLE 32

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN PLANE GEOMETRIES PUBLISHED ABOUT 1900

Textbook No.	Arithmetic	Algebra			Geometry			Trigonometry
		N. & L.	Graph	Total	Plane	Solid	Total	
81	0.1	8.1	...	8.1	91.1	0.6	91.7	...
82	4.6	7.4	...	7.4	87.6	0.4	88.0	...
83	2.4	7.4	...	7.4	90.1	0.2	90.3	...
84	1.0	11.9	...	11.9	86.6	0.6	87.2	...
85	1.0	9.1	...	9.1	89.7	0.2	89.9	...
Mean ..	1.8	8.8	...	8.8	89.0	0.4	89.4	...

still found in nearly every textbook. There was a small amount of arithmetic which was used chiefly in the solutions of problems. No trigonometry was found in any book. The textbooks of the nineteenth century were very much like those at the beginning of the present century. Table 31 shows two books of somewhat

different types. Book number 79 was one in which the work was almost all pure geometry and number 80 one in which there was a considerable portion of algebra, especially if the work is viewed in the light of the geometry course of the past century.

2. *Effect of the Correlated Mathematics Movement on the Plane Geometry Textbooks.*—Between 1900 and 1915, there was a considerable change with respect to general mathematics in the geometry textbooks. (See Tables 32 and 33.) This was the period of time in which correlated mathematics was being advocated most ardently. The amount of plane geometry was reduced from 89.0 per cent to 81.8 per cent. This made way for a small increase in algebra, an increase in arithmetic from 1.8 per cent to 5.7 per cent, and the introduction of a little numerical trigonometry.

TABLE 33

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN PLANE GEOMETRIES PUBLISHED ABOUT 1915

Textbook No.	Arithmetic	Algebra			Geometry			Trigonometry
		N. & L.	Graph	Total	Plane	Solid	Total	
86	7.7	10.3	...	10.3	81.5	0.5	82.0	...
87	6.3	13.6	...	13.6	78.6	...	78.6	1.5
88	6.0	11.6	1.5	13.1	78.2	0.8	79.0	1.8
89	3.9	7.3	...	7.3	86.5	0.2	86.7	2.2
90	4.4	9.8	...	9.8	84.3	0.2	84.5	1.3
Mean ..	5.7	10.5	0.3	10.8	81.8	0.3	82.2	1.4

3. *The Extent of General Mathematics in the Plane Geometry Textbooks of To-day.*—Very little change in the subject matter of the textbooks between 1915 and 1928 was noticeable in respect to general mathematics. The average of the textbooks of 1928 did not differ from that of 1915 for any subject by more than 0.6 per cent of the pages. (See Tables 33 and 34.) This is a very small difference and it shows that there has been no influence during the past thirteen years which has caused the plane geometry course to be appreciably changed.

4. *Subject Matter in the General Mathematics Textbooks.*—The subject matter in the tenth grade general mathematics textbooks

was much more general than in the plane geometry textbooks, as may be seen from a careful study of Tables 34 and 35.

Only 70.6 per cent of the work was geometry instead of 82.8 per cent as in the plane geometries of to-day. Of this 6 per cent was solid geometry, most of which was demonstrative solid geometry. More than 17 per cent of one of the six general mathematics books examined was solid geometry. An average of about one-fifth of the work was algebra. A considerable portion of the algebra, however, was used in algebraic proofs of geometric theorems. In this way, demonstrative geometry was advanced perhaps as much as though the algebra were omitted. Besides aiding in the proofs of theorems, the algebra already learned was in this way constantly reviewed. A much greater portion of trigonometry and a somewhat smaller portion of arithmetic was contained in this type of textbook than in the plane geometries.

TABLE 34

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN PLANE GEOMETRIES PUBLISHED SINCE 1923

Textbook No.	Arithmetic	Algebra			Geometry			Trigonometry
		N. & L.	Graph	Total	Plane	Solid	Total	
91	3.6	12.2	...	12.2	82.4	0.2	82.6	1.7
92	5.8	8.9	0.5	9.4	83.9	...	84.9	...
93	5.7	7.7	...	7.7	84.3	1.0	85.3	1.3
94	5.5	10.1	...	10.1	84.2	0.2	84.4	...
95	9.1	11.1	...	11.1	78.0	0.5	78.5	1.3
96	6.9	9.9	...	9.9	80.2	0.2	80.4	2.8
97	4.7	15.1	0.4	15.5	78.2	0.4	78.6	1.3
98	2.5	13.2	...	13.2	84.1	0.2	84.3	...
99	2.7	9.6	...	9.6	87.7	...	87.7	...
100	4.0	13.6	...	13.6	79.5	1.6	81.1	1.3
Mean ..	5.1	11.1	0.1	11.2	82.3	0.4	82.8	1.0

RELIABILITY OF PER CENTS IN CORRELATION TABLES

A study was made of the reliability of the per cents in the correlation tables with the same four textbooks as was done with

respect to the distribution of subject matter. In the two independent investigations made of these four books, it was found that two of the correlation per cents differed about 4 per cent, five about 3 per cent, two about 2 per cent, seven about 1 per cent, and eight not more than 0.5 per cent. Inasmuch as these four textbooks were among the most difficult to investigate, these figures indicate that the results were fairly reliable.

TABLE 35

PER CENT OF THE DIFFERENT BRANCHES OF MATHEMATICS IN SIX TENTH GRADE GENERAL MATHEMATICS TEXTBOOKS OF THE PRESENT CENTURY

Textbook No.	Arithmetic	Algebra			Geometry			Trigonometry
		N. & L.	Graph	Total	Plane	Solid	Total	
101	1.3	24.2	1.8	26.0	65.1	....	65.1	7.6
102	3.4	26.5	0.9	27.4	54.1	8.8	62.9	6.3
103	0.9	15.3	2.1	17.4	74.3	1.3	75.6	6.0
104	2.5	13.4	2.9	16.3	66.4	8.2	74.6	6.6
105	2.7	17.0	...	17.0	57.0	17.6	74.6	5.7
106	3.4	19.1	...	19.1	70.9	....	70.9	6.6
Mean ..	2.4	19.3	1.3	20.5	64.6	6.0	70.6	6.5

CORRELATION OF MATHEMATICS IN THE TENTH GRADE

1. *Correlation of Subjects in the Plane Geometry Textbooks of the Nineteenth Century.*—There was very little correlation of subjects found in the two nineteenth century textbooks examined. This is revealed by an examination of the per cents in Table 36. One of the books showed a little correlation of algebra and geometry and the other a little correlation of arithmetic and algebra. The average proportion of the pages of these two books showing correlation was 5.3 per cent.

2. *Correlation of Subjects in the Plane Geometry Textbooks Published about 1900.*—Tables 36 and 37 reveal the fact that there was probably more than twice as much correlation of mathematical subjects in the five geometry textbooks published about 1900 as there was in the two of the nineteenth century. The average was 12.8 per cent instead of 5.3 per cent as was found in the two earlier geometries. Most of the correlation in 1900 was between algebra

and geometry. There was a little correlation between arithmetic and geometry and there was also a little between arithmetic and algebra.

3. *Effect of Correlated Mathematics upon Correlation in the Plane Geometry Textbooks.*—The correlated mathematics movement had a greater effect upon the amount of correlation found in the plane geometry textbooks than it did in broadening the subject matter. Between 1900 and 1915, the proportion of correlation of subjects increased from 12.8 per cent to 25.8 per cent. (See Tables 37 and 38.) The correlation between arithmetic and algebra increased from 1.8 per cent of the pages to 9.4 per cent, between arithmetic and geometry from 3.6 per cent to 14.4 per cent, and between

TABLE 36

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS WERE CORRELATED IN TWO PLANE GEOMETRY TEXTBOOKS PUBLISHED IN NINETEENTH CENTURY

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Algebra-Trigonometry	Geometry-Trigonometry
79	4.6	...	...	4.6	...	...
80	6.0	5.2	...	0.8	...	...

TABLE 37

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS ARE CORRELATED IN PLANE GEOMETRY TEXTBOOKS PUBLISHED ABOUT 1900

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Algebra-Trigonometry	Geometry-Trigonometry
81	7.6	0.4	0.4	7.6	...	...
82	12.4	2.1	6.8	6.8	...	...
83	11.7	3.3	5.8	9.2	...	...
84	20.2	1.6	2.5	19.3	...	...
85	12.1	1.7	2.5	11.3	...	...
Mean	12.8	1.8	3.6	10.8	...	...

TABLE 38

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS ARE CORRELATED IN PLANE GEOMETRY TEXTBOOKS PUBLISHED ABOUT 1915

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Algebra-Trigonometry	Geometry-Trigonometry
86	28.0	8.2	18.1	18.1	...	...
87	30.9	12.6	16.8	26.7	...	...
88	29.8	12.0	16.0	25.8	...	...
89	18.1	6.2	9.8	14.5	...	...
90	22.3	7.9	11.5	18.0	0.3	0.3
Mean	25.8	9.4	14.4	20.6	0.1	0.1

algebra and geometry from 10.8 per cent to 20.6 per cent. These figures represent a considerable increase in the amount of fusing of the subject matter.

4. *Correlation of Subjects in the Plane Geometry Textbooks of To-day.*—Tables 38 and 39 show that the examination of textbooks revealed practically no change from 1915 to 1928 in the per cent of the pages on which the mathematical subjects are correlated in the tenth grade textbooks. It has already been shown that the general character of the textbooks has been stationary for the past thirteen years. The same thing may be said in regard to the changes in the amount of correlation. The statement has often been made that geometry is the most nearly static course in mathematics in the high school curriculum. Certainly the present investigation of high school geometry would corroborate such a statement.

5. *Correlation of Subjects in the Tenth Grade General Mathematics Textbooks.*—The average amount of correlation of subjects in the six general mathematics books examined was not much greater than in the plane geometries of to-day. Table 40 shows a total correlation of 29.5 per cent of the pages in the general mathematics textbooks, while Table 39 shows that 25.5 per cent of the pages in the recently published geometry books treated two or more subjects. There was less correlation of arithmetic and algebra, less of arithmetic and geometry, and more of algebra and geometry

TABLE 39

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS ARE CORRELATED IN PLANE GEOMETRY TEXTBOOKS PUBLISHED SINCE 1923

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Algebra-Trigonometry	Geometry-Trigonometry
91	25.8	8.3	9.5	23.6	...	0.3
92	22.3	10.1	14.5	17.3	0.3	0.3
93	20.4	9.2	14.5	15.1	...	...
94	23.9	11.0	14.7	20.2	...	...
95	31.7	10.6	21.8	20.5	...	...
96	26.2	10.1	14.8	18.9	...	...
97	30.6	7.9	11.9	26.3	0.5	0.3
98	23.0	7.0	7.3	22.7	...	...
99	20.4	6.9	7.8	19.6	...	...
100	28.3	11.7	12.0	27.6	0.5	0.5
Mean	25.3	9.3	12.9	21.2	0.1	0.1

in the general mathematics books than in the plane geometry books of to-day.

TRENDS IN THE PLANE GEOMETRY TEXTBOOKS

It is difficult to detect any trends in the plane geometry textbooks either with respect to the general nature of the subject matter or with respect to correlation. Changes have been noted which were made in the past, but there are no significant changes being made at the present time. Many teachers think that algebra and demonstrative geometry will not mix. There is evidently some truth in this contention. Even the books written by authors who advocate general mathematics are not quite so general in character for the tenth grade as for the ninth grade. Perhaps the authors were dominated somewhat by the traditional demonstrative geometry requirement and would have made the subject matter more general if they could have done as they wished. Algebraic methods can easily be used in many of the geometric proofs. Considerable changes were made in this respect between 1900 and 1915, and perhaps more progress of this kind could be made with profit. If there are any changes being introduced at

TABLE 40

PER CENT OF TOTAL PAGES CORRELATED AND OF THOSE ON WHICH TWO PARTICULAR SUBJECTS ARE CORRELATED IN SIX TENTH GRADE GENERAL MATHEMATICS TEXTBOOKS PUBLISHED DURING THE PRESENT CENTURY

Textbook No.	Correlated	Arithmetic-Algebra	Arithmetic-Geometry	Algebra-Geometry	Algebra-Trigonometry	Geometry-Trigonometry
101	38.3	0.7	1.4	36.6	0.2	0.2
102	31.7	6.6	4.4	27.4	2.4	2.0
103	27.1	1.8	1.8	26.4	0.4	0.4
104	18.6	5.0	3.6	16.0	3.2	3.4
105	24.4	6.9	7.6	23.6	0.1	0.1
106	37.0	8.3	6.5	35.3	2.4	2.4
Mean	29.5	4.9	4.2	27.6	1.5	1.4

present which are noticeable, they are in this direction. Certainly an effort should be made to keep the algebra fresh in mind if its study is to be resumed in the eleventh grade.

GENERAL CONCLUSIONS FROM THE STUDY OF TEXTBOOKS

There has been no remarkable change in the textbooks for the seventh and eighth grades, ninth grade, or tenth grade at any particular period of time since 1900. In the seventh, eighth, and ninth grades, a gradual change has been taking place. This change has been away from the tendency of the past to confine the year's work to one subject alone, and toward what is commonly called general mathematics. One of the influences which has contributed to this change is the junior high school. Many educators think that the work in the junior high school should be of an exploratory nature. This naturally leads to a greater variety of subject matter. Certainly the best part of mathematics should be given to the pupils, regardless of what subject or subjects it is. Since the junior high school is not bound by tradition, textbook writers have been able to introduce whatever they think is best. The only one of the four grades in which the subject matter in the textbooks is static is the tenth grade. No change is apparent in this grade.

There is no indication in any four of the grades of an increase in the amount of correlation or mixing of the different branches of mathematics. If any increase occurs, it will probably be due to the possibility of greater correlation caused by a more general type of mathematics being introduced, and consequently more material that can be correlated.

CHAPTER VIII

EXTENT OF THE TEACHING OF GENERAL MATHEMATICS

A progressive teacher of mathematics told the writer at the annual meeting of the National Council of Teachers of Mathematics in Boston, in February, 1928, that a very small per cent of the pupils in the American high schools were taught general mathematics. In a recent article, an equally progressive teacher said: "To-day the pupils enrolled in courses in general mathematics are numbered by the hundred thousands." The question naturally arises as to which statement is correct. The difference may have been due to misinformation by one or both of these teachers as to how extensively general mathematics is taught or to a different interpretation of what constitutes general mathematics. Some data have been published, which indicate how these differences in opinion may have arisen. Some of these data are misleading and do not give a true interpretation of the facts, as will be shown in the present chapter.

The writer made an investigation in July, 1928, of all of the *printed* courses of study available at the Bureau of Curriculum Research, Teachers College, Columbia University. There were a large number of mimeographed courses of study in the files of the Bureau, which he did not investigate. As a general rule, the mimeographed courses are not so carefully worked out as the printed ones. Because of this fact and because they are so numerous, they were not included in the study made. In January, 1929, the writer sent to the board of education in each of the cities and states included in this investigation for new courses of study so that the investigation might represent the most recent revisions which it was possible to obtain. Replies were received from all but three cities and one state. A report of the findings of the investigation of the most recent courses of study will be given in this chapter.

DATA CONCERNING THE EXTENT OF GENERAL MATHEMATICS

Data are available which would seem to indicate the extent to which general mathematics is being taught. These data are the published findings of three different studies.

1. *Per Cent of Junior High Schools Offering Trigonometry and Demonstrative Geometry in the Ninth Grade.*—In 1925, E. H. Worthington, of Temple University, Philadelphia, sent out questionnaires to a large number of junior high schools in Pennsylvania and in other states.[1] Among other things he sought information as to whether trigonometry and demonstrative geometry were taught in the respective schools. Replies from eighteen schools in Pennsylvania and from thirty-three in sixteen other states were tabulated. According to the replies, 41 per cent of the schools included some trigonometry in the ninth grade course, and 31 per cent included some demonstrative geometry. This per cent of demonstrative geometry might be as much as ten points too high according to Worthington, because of a possible confusing of intuitive geometry with demonstrative geometry.

2. *Program of Studies in Junior High Schools.*—A report was made early in 1927 of certain facts gleaned from bulletins containing courses of study in seventy-eight junior high schools.[2] These bulletins were collected in 1925–26. The investigators sent requests for bulletins to superintendents of schools in three hundred leading high school centers throughout the country. About one-half of the superintendents replied, but comparative data from only seventy-eight bulletins could be used. Table 41 gives the results of the part of this investigation which pertains to mathematics. The constant courses represent the work required of all pupils in the grade, and the variables represent elective courses.

The names of subjects were set down exactly as they appeared in the original programs. Hence it is quite probable that certain subjects given under different names were either alike or quite similar. An examination of Table 41 reveals that the majority of the courses were called "Mathematics," without specifying the particular subject or subjects included. This designation of courses is in harmony with the titles of junior high school text-

[1] E. H. Worthington, "Current Practice in Junior High School Mathematics," *Mathematics Teacher*, 19:72-76 (1925).

[2] R. M. Tryon, H. L. Smith, and A. F. Rood, "The Program of Studies in Seventy-Eight Junior High School Centers," *School Review*, 35:96-107 (1927).

TABLE 41

NUMBER OF JUNIOR HIGH SCHOOL CENTERS OFFERING MATHEMATICS

Course	Seventh Grade		Eighth Grade		Ninth Grade	
	Constant	Variable	Constant	Variable	Constant	Variable
Mathematics* ...	78	0	76	5	45	35
1. Mathematics	50	0	54	3	34	13
2. General Mathematics	3	0	5	0	2	5
3. Arithmetic ..	25	0	17	2	1	3
4. Elementary Geometry ...	1	0	0	0	0	0
5. Algebra	0	0	2	0	19	21
6. Practical Mathematics	0	0	0	0	0	1

* The figures opposite the general heading, "Mathematics," do not represent totals for the subdivisions, since a given school may offer more than one type of mathematics.

books. The books are not called arithmetic, algebra, or general mathematics, but are commonly called by such titles as "Junior High School Mathematics," "Modern Junior Mathematics," "Junior Mathematics," and so on. All but two or three of the junior high school textbooks are good examples of general mathematics. In keeping with these titles, it is reasonable to assume that the courses which were called "Mathematics" were general mathematics. If this is true, the constant mathematics courses would be divided between general and traditional mathematics as follows: 53 schools offered general mathematics and 26, traditional mathematics in the seventh grade; 59, general mathematics and 19, traditional mathematics in the eight grade; and 36, general mathematics and 20, traditional mathematics in the ninth grade. The variable courses were confined almost entirely to the ninth grade in which there were 18 courses in general mathematics and 25 courses in traditional mathematics. Nearly all of the so-called traditional courses in the ninth grade were called "Algebra." In two of the junior high school series of textbooks, the seventh and eighth grade books are called "Mathematics" and the ninth grade books, "Algebra," even though both of the books in the ninth grade are very good examples of general mathematics.

Hence it is natural to suppose that the courses of study bulletins would follow somewhat the examples of those two authors if their textbooks were used. It is quite probable, however, that there are more junior high schools offering traditional algebra in the ninth grade than traditional arithmetic in the seventh and eighth grades.

3. *Program of Studies in Senior High Schools.*—Another report was made in 1927 of an analysis of curricula.[3] C. V. Good, of Miami University, Oxford, Ohio, examined the curricula of thirty senior high schools located in the largest cities of various states and distributed over a wide geographical area. Table 42 gives the results in regard to ninth and tenth grade mathematics.

TABLE 42

MATHEMATICS COURSES OFFERED IN THIRTY SENIOR HIGH SCHOOLS

Subject	Ninth Grade		Tenth Grade	
	Constant	Variable	Constant	Variable
Mathematics*	18	21	4	36
1. Algebra	10	9	1	7
2. Arithmetic	0	4	0	1
3. Elementary Algebra	0	1	0	0
4. General Mathematics	2	3	0	2
5. Geometry	0	0	2	10
6. Mathematics	6	2	1	5
7. Plane Geometry	0	0	0	8
8. Rapid Calculation	0	0	0	1
9. Shop Mathematics	0	0	0	2
10. Special Mathematics	0	1	0	0
11. Vocational Mathematics	0	1	0	0

* The figures opposite the group heading, "Mathematics," represent the total of the subdivisions.

The table shows that about 13 per cent of the total number of ninth grade courses and about 5 per cent of the tenth grade courses were general mathematics. The term "Mathematics" under subdivision number 6 is indefinite. It is probable that these "mathematics" courses were for the most part general mathematics. The number of courses called "mathematics" without designating

[3] C. V. Good, "The Mathematics and Science Curricula in Junior and Senior High Schools," *School Science and Mathematics*, 27:863-69 (1927).

the kind was relatively smaller, however, in the senior high schools than in the junior high schools. This is to be expected since not so many ninth grade textbooks are designated "mathematics" without the type being specified. A thorough examination of the complete courses of study or of the textbooks used would be the only safe criterion as to the kind of mathematics offered. Even then there would be differences of opinion in many cases, due to an overlapping of the two types. The table shows one indisputable fact; namely, that general mathematics is more popular in the ninth grade than in the tenth grade. The analysis of textbooks also conclusively revealed this fact. So far as preparation for college is concerned, however, the results of Professor Bussey's study (see page 40) indicate that it is not desirable to offer general mathematics in the ninth grade and traditional mathematics in the tenth grade.

COURSES OF STUDY

For the purposes of this investigation, the courses of study have been divided into three classes: city courses, state courses, and syllabi of the College Entrance Examination Board. In 1928, there were printed courses of study in mathematics in the files of the Bureau of Curriculum Research at Teachers College, Columbia University, from thirteen cities and eight states. To these were added for the present investigation, courses for the state of Minnesota and the College Entrance Examination Board which the writer had in his possession. In every instance in which a more recent bulletin was obtained, it was used instead of the one on file at Teachers College. A complete list of the bulletins used is given at the end of this chapter.

TYPE OF MATHEMATICS IN THE COURSES OF STUDY

There is a danger that in this discussion of courses of study, a judgment of the type of mathematics will be based upon a wide variety of subject matter rather than upon a choice of the best mathematics available for the pupils and a psychological arrangement. In the seventh and eighth grades, the older arithmetics included some intuitive geometry under the name of mensuration. Hence the fact that there are two subjects, arithmetic and the geometry of mensuration, included in the older arithmetics in the work of these two years will not differentiate general from traditional mathematics. The National Committee recommended

that about two-thirds of the ninth grade mathematics be algebra, including numerical trigonometry, and the remainder an introduction to demonstrative geometry. John A. Swensen of the Wadleigh High School in New York City has commented on the misconceptions of many authors in regard to the committee's recommendations of ninth grade textbooks in the following statement:[4]

> In spite of these very plainly worded recommendations [in the National Committee's report], many authors of old-fashioned algebras with a few pages devoted to numerical trigonometry, but no demonstrative geometry whatsoever, have boldly announced in the preface: "This book follows the recommendations of the National Committee for the ninth grade."

The present writer believes that such a ninth grade course represents a modification of the stand for traditional algebra, but that the inclusion of a little trigonometry does not make it general mathematics. In judging the type of course, he has tried to determine the general spirit of the whole course and not merely whether two, three, or even four subjects are included in it. A course, to be general mathematics, must contain at least two branches of mathematics in the year's work.

CITY COURSES OF STUDY

1. *Fresno, California, 1928.*—The course of study bulletin outlines a three-year course in junior high school mathematics, a year course in elementary algebra, a year course in plane geometry, and other more advanced courses. In the discussion of junior high school mathematics, the following statement is made with respect to general mathematics:

> If we can redistribute the materials of our mathematics courses so that the ideas may have more time to grow in the minds of our pupils they will be able to meet, in a more efficient way, the demands of our complex daily life.
>
> We believe that with sensible restrictions, a course in general mathematics will best meet the demands of modern life and that our girls and boys in the junior high school should have exploratory advantages in this course, for the sake of interest, information and the development of power, and self-confidence.

The outline of work for the junior high school is based upon a series of textbooks which are of the general mathematics type. In the seventh grade, arithmetic, algebra, intuitive geometry, and statistical graphs have a place. The work for the eighth grade has a similar distribution of subject matter. The work for the

[4] John A. Swensen, "The Newer Type of Mathematics," *Mathematics Teacher*, 22:152-55 (1929).

ninth grade centers around algebra with some arithmetic, numerical trigonometry, and applications of geometry introduced. Apparently the course maintains the spirit of general mathematics throughout.

The outline of the course in elementary algebra for the ninth grade in the four-year high school is much more of the traditional type. The use of formulas is placed early in the course. The remainder of the outline is pure algebra until near the end, where a unit of numerical trigonometry is introduced. This would encourage the omission of trigonometry more than if it were placed earlier, as is done in the junior high school course. The course of study with its objectives introduces three of the branches of mathematics during the freshman year; namely, algebra, intuitive geometry, and trigonometry. A teacher could follow this prescribed course and teach an excellent type of general mathematics or he could, on the other hand, make it much like the traditional course in algebra.

The tenth grade course is plane geometry with an introduction of "Intuitive and Constructional Geometry." It is the traditional subject matter for this grade, but shows the modern spirit in attempting to make the introduction easier.

2. *Los Angeles, California, 1928.*—The outline for plane geometry follows the adopted text and is the traditional course in geometry for the tenth grade.

3. *Oakland, California, 1925.*—The Oakland Public Schools publish two high school mathematics courses of study bulletins, one for the junior high school and the other for the senior high school. Both the seventh and eighth grade courses include arithmetic, intuitive geometry, and statistical graphs. The ninth grade course for the junior high school is chiefly algebra, but includes a unit of numerical trigonometry and some statistical graphs. The ninth grade course outlined in the senior high school bulletin is the same as the one for the junior high school.

The plane geometry course for the tenth grade is planned so that either a year of plane geometry or a year of plane and solid geometry may be given. If the course in plane and solid geometry is offered, three-fourths of it is plane geometry and the remainder is the mensuration of solids, including the development of standard formulas for areas and volumes of solids and numerical computations involving these formulas. Although the outline indicates a

course somewhat along traditional lines, the foreword is quite significant of a newer trend. It is in part as follows:

> The aim has been to select and arrange the material on a psychological rather than a strictly logical basis. In general, the changes are characterized by an early introduction of certain subjects that have formerly been reserved for advanced students, by the extension of electives upward, by the lowering of the fences between the different fields of mathematical work, by the elimination of purely artificial difficulties, and by the consistent emphasis on the applications in the world outside of school. While the core of each term's work is a separate subject, everyone must be made conscious of the essential unity in all mathematics work. Let him use arithmetic and geometry and trigonometry in the elementary algebra; let him use arithmetic and algebra and trigonometry throughout the geometric work. In this way mathematical education may be a process of continuous, consistent growth and development.

4. *San Francisco, California, 1927, 1928.*—The junior high school course of study bulletin (1927) outlines a course in general mathematics for the seventh and eighth grades with a junior high school general mathematics textbook as the basic text. The ninth grade outline provides for an option between general mathematics and traditional mathematics. The seventh grade work includes arithmetic, intuitive geometry, and graphs. The eighth grade course includes arithmetic, algebra, intuitive geometry, and a small amount of graphs and trigonometry. The ninth grade algebra outline is pure algebra with the exception of a small unit of numerical trigonometry included in the course. The ninth grade general mathematics course is outlined as follows:

GRADE LOW NINE

Unit One—Essentials of arithmetic (five weeks)
Unit Two—Ways of showing number relations and equations (five weeks)
Unit Three—Measurements of lines, areas, and volumes (five weeks)
Unit Four A—Industrial applications for boys (five weeks)
Unit Four B—Vocational applications for girls (five weeks)

GRADE HIGH NINE

Unit One—Simple applications of trigonometry (five weeks)
Unit Two—Variation and proportion (four weeks)
Unit Three—Elementary statistics (three weeks)
Unit Four—Mathematics of the business of everyday life (eight weeks)

The purpose of the two courses is explained in the introduction in these words: "The algebra is designed for those pupils who will continue the study of mathematics in the senior high school, and the general mathematics, for those who will not, and who can

therefore gain more by a course less specialized than is algebra." This contention that algebra is a better preparation for advanced mathematics represents the popular belief, but it is not supported by experimental evidence. General mathematics has been shown to be at least as good a preparation for higher mathematics as is traditional mathematics. (See Chapter IV.) The type of general mathematics which is outlined for the San Francisco schools may not be as good preparation for higher mathematics, however, because it includes a very small amount of algebra.

The senior high school Mathematics Curriculum Bulletin (1928) suggests that the pupils be separated into three ability groups in the ninth and tenth grades. It suggests that the group with the lowest ability in the ninth grade be given a course in introductory mathematics, and the other two groups a course in algebra. The algebra course is like the one for the ninth grade in the junior high school bulletin. The introductory mathematics course provides for twenty weeks of arithmetic, ten weeks of algebra, and ten weeks of intuitive geometry with algebraic formulas. No provision is made for different types of work for the different ability groups in the tenth grade. Apparently, it is assumed that the pupils of low ability will not take tenth grade mathematics. Plane geometry with a little trigonometry included is outlined for this year.

5. *Denver, Colorado, 1926, 1927.*—The junior high school program (1926) is much like general mathematics courses for the seventh and eighth grades in other cities. A little numerical trigonometry is included in the eighth grade outline. There is considerable correlation of algebra and intuitive geometry in the ninth grade. Algebra is the core of this year's work. Geometry is used to assist in the teaching of algebra wherever it can be applied. Numerical trigonometry and elementary statistics are included in the year's work.

The tenth grade course (1927) combines plane and solid geometry. In this respect, it is like the Oakland, California, course. The argument given for including solid geometry is that the pupils may get some knowledge of three-dimensional space while in high school.

6. *Boston, Massachusetts, 1923, 1925.*—The course of study for the ninth grade was published in 1923 and is being revised at the present time (1929). It consists of eight divisions, all of which

have common algebraic headings. The subdivisions under two of the major headings emphasize geometry. Under the heading, "Problems Leading to Easy Equations," the following geometric sources are suggested: complementary angles, supplementary angles, sum of angles of a triangle, angles of a stripe, and ratios of a line segment, of areas, and of volumes. Under the heading, "Evaluation," the formulas of triangles, parallelograms, rhomboids, trapezoids, and circles are suggested. The bulletin recommends that the figures be constructed and the properties studied.

The tenth grade course, published in 1925, is much more like general mathematics than the usual course in plane geometry. In fact, the following excerpt seems to indicate a typical general mathematics course:

> The mathematics of Grade X is largely plane geometry. It also includes some numerical trigonometry, and a further study of measurement, computation, and the algebraic equation with its use in the solution of geometric problems and in the demonstration of theorems.
>
> The work in trigonometry gives an opportunity for much computation of a practical sort, most of which should be done with three-figure numbers.
>
> Many theorems in geometry can be used to furnish material for the review of numerical equations. Among such theorems are those dealing with complementary angles, supplementary angles, vertical angles, angles of a stripe, angles of a triangle, perimeters and areas of plane figures, similar triangles, the Pythagorean theorem, the secants, chords, and tangents of circles.
>
> Literal equations naturally arise from the use of single letters to represent geometric magnitudes, and from the use of these letters in proofs of theorems.
>
> There should be enough simple work with functions to enable the pupils to make the necessary transformations of the literal equations he needs to use in demonstration, and of numerical equations he needs to use in the solution of problems.

7. *Detroit, Michigan, 1926.*—The Detroit course of study for the seventh grade and the first half of the eighth grade is almost entirely arithmetic. In fact, it bears little resemblance to a general mathematics course. The last half of the eighth grade has intuitive geometry as its core, but there is considerable fusing of algebra and geometry throughout the work provided for this half-year. A general mathematics textbook is used as a basic text beginning with the last half of the eighth grade and continuing throughout the ninth grade. Most of the work of these one and one-half years of mathematics is a fusion of algebra and geometry. The comments made in regard to ninth grade mathematics indicate the type of course outlined.

The course outlined for the ninth grade, together with the seventh and eighth grade courses, includes most of the fundamental mathematical notions and devices which can be taught to children of intermediate school age and maturity. The course provides a foundation for future mathematics, as well as important mathematical skills and concepts useful in everyday life. The construction of the material is such that a premium is put upon the ability to do mathematical thinking. This should, in the last analysis, be the factor determining the choice of further courses aiming at careers which demand that ability. Too often the ability to succeed in mathematical work is based upon the ability to perform certain skills. So much for the reasons for choosing a general mathematics course for the ninth grade pupils of the intermediate schools.

The above statement of reasons is in conformity with the prevailing notion that the junior high school courses should be of an exploratory nature. If they should be of this nature, then general mathematics should certainly have a permanent place in the junior high school.

8. *St. Louis, Missouri, 1926.*—The junior high school course of study in St. Louis is of the general mathematics type. The course is not called general mathematics in the junior high school bulletin, but a statement made in the tenth grade course of study implies that the pupils have had general mathematics in the ninth grade. The ninth grade course gives approximately forty objectives, about thirty of which are algebra; two, trigonometry; and eight, intuitive geometry. In terms of double pages in the course of study outline, there are about seven double pages of algebra, one double page of trigonometry, one and one-half double pages of intuitive geometry, and two double pages of review of the three years of junior high school mathematics.

The parallel arrangement of algebra and geometry which has been so popular in Europe is recommended in the St. Louis course of study for the tenth and eleventh grades. The plan provides for three days a week of geometry and two days a week of algebra in the tenth grade. The numbers are reversed in the eleventh grade. Perhaps the best way to give an idea of the plan and the reasons for it is to quote from the course of study:

This departure from the traditional block courses in secondary school mathematics has been made after careful study and investigation. Experience for a few years and in a somewhat limited number of places has shown that a course in general mathematics followed by a year of geometry and a year of algebra is not productive of the best results. . . . At present, every successful teacher spends some time in the geometry course teaching the necessary

additional algebra. The psychology of learning would seem to indicate the necessity for the redistributing of learning over a period and the reduction of the interval between the presentation of ideas of similar type, and the opinion of authorities, both in general pedagogy and in the pedagogy of secondary school mathematics, are almost unanimously in favor of teaching algebra and geometry simultaneously. . . .

. . . . . .

The organization of the traditional geometry content has not been disturbed to any great extent. The hope for improvement rests upon elimination of less important topics, and changes in sequence, procedure, and coördination. The proposed plan provides contact with the necessary propædeutic algebra under the same teacher, greater opportunity for correlation, three days as the largest intervening time between contact with geometric ideas and an opportunity for a conscious organization of the subject which furnishes an excellent review.

From the point of view of algebra, the advantage lies in the fact that the elementary algebra will be kept fresh and will be extended during the year; there is opportunity for immediate geometric application, whenever it is advisable; and the coördination of the courses in the second half of the eleventh grade will give better preparation for subsequent work in mathematics.

The parallel arrangement is not in itself a type of general mathematics. If the two subjects are kept distinct, the organization of material is not affected. If the subjects are coördinated and correlated as is suggested in the above quotation, then the courses may become general mathematics. There is one decisive difference, however, and that is that the arbitrary separation into three days of geometry and two of algebra each week breaks into the unity of the course. The parallel plan of teaching these two subjects was tried in a number of places early in the present century and so far as the writer has been able to ascertain was finally abandoned in every instance. If the course of study is used as a suggested distribution of the work and teachers are permitted extreme liberty in organizing the material, the plan might prove quite helpful.

9. *Bayonne, New Jersey, 1924.*—Algebra with a unit of numerical trigonometry comprises the ninth grade course of study. There are seventeen main divisions in the course. One of them is numerical trigonometry, another is mainly statistical graphs, and a third emphasizes geometric applications in connection with formulas. All of the remainder is apparently pure algebra. The tenth grade course is the traditional one in plane geometry.

10. *Trenton, New Jersey, 1925.*—The junior high school course of study aims to conform to the suggestions of the National Com-

mittee on Mathematical Requirements. The outlines for each year are fairly good examples of general mathematics. The program differs somewhat for average, slow, and advanced groups. The courses for the ninth grade for the average group consists of fifteen parts as follows: (1) the formula including arithmetical and geometric formula; (2) equations with applications from geometry; (3) statistical and algebraic graphs; (4) positive and negative numbers; (5) intuitive geometry; (6) algebraic technique; (7) arithmetic; (8) history and biography; (9) formula; (10) equation; (11) the algebraic graph; (12) algebraic technique; (13) arithmetic; (14) history and biography; and (15) numerical trigonometry.

11. *Philadelphia, Pennsylvania, 1923, 1926.*—The course of study bulletin for junior high school mathematics published in 1926 contains general seventh and eighth grade junior high school courses and two courses for the ninth grade. One ninth grade course is in algebra and the other in general mathematics. The algebra course is almost pure algebra, no reference being made even to a unit of numerical trigonometry. The course in general mathematics is divided into thirteen parts. Seven of these parts are chiefly algebra, one is graphs and statistics, two are intuitive geometry, two are numerical trigonometry, and one is demonstrative geometry. The general mathematics course is recommended for those pupils who do not expect to complete a high school course. In the bulletin for 1923, the same courses in algebra and general mathematics are listed for the ninth grade as are listed in the junior high school bulletin and the traditional course in plane geometry for the tenth grade is listed.

12. *Austin, Texas, 1925.*—Most of the public schools in Texas are divided according to the 7–4 plan; that is, there are seven years in the elementary school and four years in the high school, the eighth grade being the first year of high school. The courses in mathematics in the Austin high schools are called algebra, according to the 1925 bulletin. The first half of the eighth grade work is correlation of arithmetic and algebra. The next year and a half is almost all pure algebra.

13. *Dallas, Texas, 1928.*—The course of study consists of a year and a half of algebra including two weeks of trigonometry, followed by a year's course in plane geometry. There is no tendency shown to adopt any of the principles of general mathematics.

STATE COURSES OF STUDY

1. *Florida, 1924.*—Three programs of study are submitted in the course of study bulletin for the Florida schools. They are arranged graphically in Table 43.

TABLE 43

GRAPHIC ARRANGEMENT OF MATHEMATICS CURRICULUM FOR THE STATE OF FLORIDA

Grade	6–3–3 Plan	8–4 Plan	Required only through ninth grade
Seventh	Arithmetic Intuitive Geometry	Arithmetic	Correlated Mathematics
Eighth	Algebra—four fundamental operations. Applied Arithmetic	Arithmetic	Correlated Mathematics
Ninth	Algebra—special products, factoring, etc., to quadratics Numerical Trigonometry Arithmetic—Business applications	Algebra to quadratics	Correlated Mathematics
Tenth	Algebra—quadratics, exponents, radicals, logarithms, progressions, binomial theorem Plane Geometry: Books I and II	Algebra—quadratics, exponents, radicals, logarithms, progressions, binomial theorem Plane Geometry: Books I and II	
Eleventh	Plane Geometry: Books III, IV, and V Solid Geometry	Plane Geometry: Books III, IV, and V Solid Geometry	
Twelfth	Advanced Algebra Trigonometry	Advanced Algebra Trigonometry	

The course of study for the ninth grade in the 6-3-3 plan provides for offering an introduction to demonstrative geometry instead of the arithmetic as provided for in the graphic arrangement. The following comment, however, is made concerning its substitution:

> It is probably true that in Florida we shall not be able to include demonstrative geometry in our ninth grade work until the junior high school has been in operation for a few years and the students have had good courses in intuitive geometry. Perhaps for this reason it would be well that the business arithmetic be given one-third of the time, for the present, at least, rather than the demonstrative geometry.

The ninth grade course for schools organized on the 6-3-3 plan is really general mathematics with no emphasis upon correlation. Teachers may present different subjects during the year and these are kept more or less distinct if so desired. The courses for schools organized on the 8-4 plan are of the traditional type.

The outline of the course in correlated (general) mathematics for the ninth grade is divided into fifteen divisions as follows: (1) review of fundamental processes of arithmetic; (2) the straight line; (3) the equation; (4) angles; (5) the equation applied to areas and volumes; (6) the equation applied to angle pairs; (7) the equation applied to the triangle; (8) positive and negative numbers; (9) graphs including the frequency curve, graphs of linear equations, graphs of functions, graphs of formula, and graphs of variation; (10) similar triangles and proportion; (11) indirect measurement, similar figures, and trigonometry; (12) simultaneous linear equations; (13) roots and powers; (14) logarithms; and (15) quadratic equations. The following is part of the comments made concerning correlated mathematics:

> In high schools that require only one year of mathematics, that year should be correlated or general mathematics, putting the emphasis on algebra, if the teacher so desires. This year of correlated mathematics should be preceded by correlated mathematics in the seventh and eighth grades.
>
> The course in correlated mathematics will require a real teacher, but its handling will be a joy to any lover of mathematics, for in this year's work he will be putting the finishing touches on the mathematics course of many students, and will have the privilege of giving them those things which will be of most use to them in their life after leaving school.
>
> By the introduction of algebra, geometry, and trigonometry a wider field of mathematical methods is covered. This course will meet the needs of those who leave school at the end of the ninth year, and will enable other students to determine whether they wish to elect mathematics in the other years in high

school. The algebra, geometry, and trigonometry should be correlated as much as possible.

It would seem that if general mathematics in the ninth grade is better for those schools which require only one year of mathematics, it would also be better for those which require two or more years. It is quite evident that the makers of the Florida courses were influenced by the prevalent erroneous idea that the traditional arrangement is the type essential for preparation for college.

2. *Kansas, 1923.*—The course for the ninth grade in Kansas consists of algebra, with the suggestion that some drill be given in arithmetic if time can be found for it. Neither intuitive geometry nor numerical trigonometry is mentioned in the outline.

3. *Maryland, 1925.*—The program of studies in Maryland is arranged for separating the elementary school from the high school according to a 7-4 plan. The course of study for the first three years of high school is as follows: first year, (eighth grade) first course in algebra; second year, (ninth grade) plane geometry or second course in algebra; and third year, (tenth grade) second course in algebra or plane geometry. The course of study quotes the National Committee's report in regard to general mathematics and holds up general mathematics as a probable goal for sometime in the future, with as much immediate application of this type as practicable. The following quotation is significant of the attitude of those who prepared the course of study:

> What has become of general mathematics in the course suggested above for our schools? The first year shows only a subject entitled "First Course in Algebra." The answer to this question is that for our first year [grade 8] the closest approximation to the *Report* [of the National Committee] is found in the work specified as first course in algebra, which is, substantially, a general first view of the interrelations of arithmetic, algebra, geometry [intuitive], and arithmetical trigonometry, followed by an adequate treatment of such algebraic topics and operations as are now believed to be necessary for beginning pupils, many traditional topics and processes being omitted from this year's work. This view of the first-year field has, therefore, largely absorbed the material known for the last five years as general mathematics.

4. *New York, 1928.*—The New York State Department of Education provides a course of study for junior high school mathematics, one in algebra, one in plane geometry, and certain other more advanced courses. The junior high school course was first introduced in the fall of 1928. It is an excellent example of general mathematics. The seventh and eighth grade work is typical

of the new general mathematics and conforms very well to the recommendations of the National Committee. The outline for the ninth grade consists of algebra and intuitive geometry for the first half-year, algebra being the core of the half-year's work. The last half-year contains algebra, numerical trigonometry, and demonstrative geometry. The syllabus is rather indefinite in its suggestions as to just what should be included in the unit of demonstrative geometry. There is need of considerable experimentation in regard to this part of the course. In the past, all the New York state examinations have been based upon a somewhat traditional type of mathematics. The new junior high school syllabus will probably produce far-reaching results because pupils may now take the new mathematics and be as well prepared for the state Regents Examination as if they had completed a seventh and eighth grade course in arithmetic and a ninth grade course in algebra. Because of its great population, New York State has had a greater influence upon the practice of textbook writers than any other state in the United States. In order for publishers to sell their books in New York, it has been necessary to meet the requirements of the state course of study. Hence the books used in practically every state to-day reflect to a considerable extent the New York requirements. Before 1928, this state had not provided for a course in general mathematics. The failure to provide for such a course discouraged its use in all parts of the country. Just what effect the new junior high school course may have upon the mathematics for these three grades is difficult to predict, but if the course is indicative of a growing tendency in New York to offer a general course, it may result in general mathematics being universally accepted in the junior high schools. It would then probably be only a few years until similar work would be given in the same grades in schools of the 8-4 plan. Since the movement is so significant, a rather extended quotation from the syllabus is here given:

Unified versus stratified mathematics. Until very recently the standard practice of American schools, under the 8-4 plan of organization, consisted in studying arithmetic during the first eight school years, with a very moderate admixture of some mensurational geometry. During the ninth school year arithmetic was usually dropped as a separate subject, to be replaced by exclusive study of algebra. Then, without special preparation, the young learner was confronted with demonstrative geometry during the tenth school year. This compartment system, while possessing some administrative

advantages, was objectionable in that it created a distorted point of view. Instead of developing the subject organically, it presented only fragments of the subject and for too limited a period of time. It was also completely at variance with the experience of the leading European countries. Since 1910 an increasing number of schools has been trying to replace this "stratified" form of organization by a correlated series of mathematical "units" or "blocks of work" chosen from the whole domain of elementary mathematics. Under this plan, a "unit" of arithmetic may be followed immediately by a "unit" of geometry or algebra or trigonometry, according to the preference of the individual school or author. Obviously, this is still a form of "stratified" mathematics, the only difference being that the strata have become much thinner. The new arrangement is often called "general" (or "correlated," "composite," "unified") mathematics.

One great advantage of this correlated plan of teaching mathematics is that important principles can be introduced at an earlier stage and can be kept under review for a much longer period to time.

The effectiveness of "unified" mathematics obviously depends on such factors as a wise choice of each unit, the thoroughness with which each unit is mastered, the degree to which it effects real orientation instead of confusion, and the manner in which the classroom procedure consciously and progressively clarifies and correlates the constituent elements of the curriculum. As a matter of fact, a virtually simultaneous use of the various mathematical tools (arithmetical, algebraic, geometric) appears to be natural or customary only in vocational courses in applied mathematics.

The ninth grade syllabus for the four-year high school is a course in algebra, including trigonometry. It suggested that geometric applications be made in connection with the study of formulas. The course is considerably different from the old type of algebra, but it could hardly be classed as general mathematics.

The tenth grade syllabus is plane geometry with an emphasis placed upon the formulas developed in the geometry. It places greater emphasis upon the formula than is usually done in courses of study in plane geometry.

5. *Oklahoma, 1925.*—The course of study bulletin for Oklahoma provides for a year of algebra for the ninth grade and a year of plane geometry for the tenth grade. The following excerpt expresses the attitude in regard to general mathematics:

Textbook writers in the past have been accustomed to divide the subject matter into independent water-tight compartments. They did this because the whole interest in the subject was in the process rather than the end. . . . Under the old conception of mathematical values such a proceeding was justifiable, but under the new conception it is indefensible. Algebraic manipulations have absolutely no values as ends in themselves; their only values are as means to ends.

There has recently arisen a demand for a course in general mathematics containing the elements of algebra, arithmetic, and geometry, and bringing out their relationships. In response to this demand a number of textbooks have been written. These courses as yet are not entirely successful, but no mathematics teacher's library is complete without one or more of these general mathematics textbooks. Some of them are listed elsewhere in this course.

The attainment of the ends suggested by the term "general mathematics" is far more up to the teacher than the textbook. The good teacher can develop the relationships between the various subjects in mathematics whether the textbook does or not.

6. *Minnesota, 1924.*—In a prefatory statement in the Minnesota syllabus, a distinction is made between four groups of users of mathematics, namely; the general readers, the trade group, the pre-engineering group, and the specializers. After discussing these four groups, a statement is made that these considerations point to the following:

A general introductory course running through the ninth grade, offered in all high schools and normally taken by all students. This course should be unified through the selection of problems and projects in fields where the pupils have both knowledge and potential interest, with the inclusion of such processes and conceptions as will serve "as instruments to the attainment of the ends set before them," the whole running across the divisions heretofore separating algebra, geometry, and trigonometry. This plan has the distinct advantage of meeting the needs of both those who go on to further study of mathematics and those who do not. The projects and instrumental processes and concepts will serve to discover the aptitudes and interests of the future specializers in mathematics, as well as to give to those who go no farther a "practical" knowledge of such parts of mathematics as are more likely to be applicable to their future vocations, insuring an adequate command of processes fundamental to many occupations, as well as a familiarity with such mathematical terms and concepts as will be needed for general informational or "interpretative" purposes.

. . . . . .

A course in the tenth grade—a "variable" in such curricula as the pre-engineering and mathematics—the content to include algebra, demonstrative geometry and trigonometry, selected largely on the basis of determined vocational needs and organized in accordance with the demands of psychological experience, interest and growth.

Two distinct courses of study are outlined for the ninth grade. One is a course in general mathematics consisting of algebra, intuitive geometry, and an introduction to trigonometry and demonstrative geometry. The other is a course in algebra. Likewise two courses are provided for the tenth grade; one in general mathematics and the other in plane geometry. The general

mathematics course contains algebra, demonstrative plane and solid geometry, and trigonometry. Much more attention is given to plane geometry than to any other subject.

While the prefatory statements favor general mathematics, the courses in algebra and geometry are included, no doubt, for those teachers and school systems that are not ready to accept general mathematics.

7. *Pennsylvania, 1927.*—Pennsylvania, like many other states, provides a course of study for the junior high school and a different course for the ninth grade in schools organized on the 8–4 plan. The bulletin states that the topics to be included in the course of study for the junior high school may be arranged as follows:

Seventh year: Arithmetic, particularly as applied to the home, to industry, and to the other subjects in the school curriculum. Intuitive geometry.

Eighth year: Algebra; arithmetic, particularly its social and commercial applications. Such geometry as grows naturally out of discussions of the size and shape of figures.

Ninth year: Algebra; numerical trigonometry; where possible, an introduction to demonstrative geometry, with the aim limited to learning the meaning of "demonstration."

The *purpose* and *content* of the ninth grade mathematics in the four-year high school is stated as follows: "In four-year high schools, the ninth year's work in mathematics should provide the pupil with as broad a foundation of mathematical training as possible. In particular, the course should include algebra, numerical trigonometry, and geometry, with at least an indication of the nature of a geometric demonstration." The bulletin also recommends that "about two-thirds of the ninth year be devoted to algebra and trigonometry, and about one-third to geometry." This is the same plan as for the ninth grade in the junior high school course of study. It is suggested that the course be modified to conform to a possible difference in training with respect to intuitive geometry. In regard to a change from algebra to general mathematics, the bulletin states that "in view of the advantage of the course in mathematics for the ninth year comprising the fundamental notions of algebra, intuitive geometry, and numerical trigonometry, it is confidently expected that gradually ninth year algebra will be replaced by a year of composite mathematics." The tenth grade course is the traditional one in plane demonstrative geometry.

8. *Vermont, 1928.*—In the introduction to the mathematics course of study bulletin for Vermont is the following statement in regard to traditional and general mathematics:

> The subject of mathematics in secondary schools has received exhaustive study during the last fifteen years by prominent educators and institutions throughout the country. Much actual experimentation has been carried out in typical high schools, in order to determine what courses and theories are sound and practical, and what are not feasible and satisfactory.
>
> These studies and experiments have concerned themselves with the aims of mathematical instruction, the evaluation of traditional and modified courses, and with the organization of work throughout six or four years of work.
>
> The results of this research have demonstrated that the traditional division of mathematics into mutually exclusive fields of arithmetic, algebra and geometry is seriously defective in grading, practical utility, interest and educational values.

It is also stated in the bulletin that the work of the ninth grade should give the pupil the most valuable mathematical training he is capable of receiving, without undue reference to the courses he may or may not take later. It further states that "Fortunately there appears to be no conflict of interest between pupils who subsequently go to college and those who do not." The mathematics outlined for grades seven to ten inclusive is as follows:

Grade Seven: Arithmetic including some geometry of mensuration.

Grade Eight: Arithmetic; elements of algebra; intuitive geometry; and mensuration and graphs.

Grade Nine: Composite course, chiefly algebra with either (*a*) or (*b*) according to the type of high school pursued by the student.

- *a* Intuitive geometry, graphs and numerical trigonometry. Home and vocational applications.
- *b*. Business or commercial arithmetic (for commercial students). Applications.

Grade Ten:

- *a*. Chiefly plane geometry, with related algebra and solid geometry optional. Continued work in numerical trigonometry and logarithms. (This course will be required in many programs of study and elective in others.)
- *b*. Commercial arithmetic or bookkeeping. (Required for commercial students where commercial course is offered.)
- *c*. Special applications or project work connected with vocational courses, community problems, homework, etc.

The eighth, ninth, and tenth grade work in the above outline is a good type of general mathematics. Concerning the relation of the different branches of mathematics in the ninth grade and the importance of algebra, the course of study is quite clear.

The major work of this year is algebra. Related work in other divisions of mathematics previously studied (arithmetic, intuitive geometry) should be continually introduced. New work in other fields, relatively little at first, will form an increasing proportion of the work as the year progresses. It should be definitely planned for as part of regular lesson assignments two or three times a week.

In this way, the bulletin suggests the unity of mathematics throughout the instruction of the ninth year. The bulletin further states that: "Related algebra, diagrams, and graphic work should be carried along with the general course for approximately the equivalent of one period per week." It also suggests that if intuitive geometry has been given in the eighth and ninth grades, a part or all of the course in solid geometry may be given in the tenth grade.

9. *Virginia, 1928.*—The Virginia course of study is planned for a four-year high school in school systems organized on a 7–4 plan. The courses are as follows:

Eighth Grade: Arithmetic, one-half year; algebra, one-half year.
Ninth Grade: Elementary algebra through quadratics.
Tenth Grade: Plane geometry.

These courses are of the traditional type. The only one of these grades in which as many as two subjects are provided is the eighth. The plan apparently presupposes that one of these subjects will be given in the first semester and the other in the second.

COLLEGE ENTRANCE EXAMINATION BOARD COURSES OF STUDY

The syllabi of the College Entrance Examination Board were revised in 1923. They follow to a considerable extent the recommendations of the National Committee. The ninth grade course is algebra, including numerical trigonometry. In considering the formulas, those for the volume of a sphere, the area of a trapezoid, falling bodies, amount at simple interest, and amount at compound interest are given as types. Although the course is called algebra, the algebra is correlated to a considerable extent with geometry through this suggested treatment of formulas. The course should probably be classed in a position somewhere between the traditional and the general type of mathematics, perhaps somewhat nearer to the general type. The College Entrance Examination Board also provides for a tenth grade course in plane geometry or for a year course in plane and solid geometry. Both courses are purely of the demonstrative type. If plane and

solid geometry are offered during the same year, the outline provides for about two-thirds of the year's work in plane geometry, followed by about a third of a year in solid geometry.

These courses are interesting chiefly because of their effect upon the high school mathematics of to-day. It is a well-known fact that the high school administrators are anxious to have their students prepared to enter college if they desire to do so. In order to prepare them to enter colleges dominated by the College Entrance Examination Board, it is necessary to offer the kind of courses required in the entrance examinations. Hence the courses of study prepared by this board have controlled to a large extent the high school situation. Experience and experiments have shown that the general mathematics courses provide as good preparation for these examinations as the traditional courses, but the outline appears on superficial examination to be of the traditional type. Hence high school teachers have taught this type because they thought that it gave the best preparation for these examinations.

SUMMARY AND CONCLUSIONS

A summary of the types of mathematics offered in the various cities and states (including the syllabi by the College Entrance Examination Board) is given in Table 44. The blank spaces in the table indicate either that there are no printed courses of study for the particular grade or type of school in question or that the writer was not able to obtain a bulletin. In cities in which all the schools are organized on the 6–3–3 plan, there would be no course of study for the ninth grade of a four-year high school. Likewise in cities in which all the schools are organized on the 8–4 plan, there would be no junior high school courses. In such cases, these are necessarily left blank in the table. Table 44 indicates the following facts based upon the courses investigated:

1. The prevailing type of mathematics in the ninth grade of the junior high school courses is general mathematics. Five of the eight cities from which junior high school courses of study were obtained offer only general mathematics; one, a course more nearly like general than traditional mathematics; and two, a choice between a course in general mathematics and a course somewhat more traditional in character than the general mathematics type. The three state courses in junior high school mathematics are general mathematics.

TABLE 44

TYPES OF MATHEMATICS IN THE COURSES OF STUDY IN THE HIGH SCHOOL BULLETINS*

City or State	Ninth Grade		Tenth Grade
	Junior High School	Four-year High School	Senior High School
Fresno, California	G	GT	TG
Los Angeles, California	.....	.....	T
Oakland, California	GT	GT	TG
San Francisco, California	G or TG	TG (G†)	TG
Denver, Colorado	G	.....	TG
Boston, Massachusetts	.....	TG	G
Detroit, Michigan	G	.....	.....
St. Louis, Missouri	G	.....	TG
Bayonne, New Jersey	.....	TG	T
Trenton, New Jersey	G	.....	.....
Philadelphia, Pennsylvania	G or T	.....	T
Austin, Texas	.....	T	.....
Dallas, Texas	.....	T	T
Florida	G	T or G	T
Kansas	.....	T	.....
Maryland	.....	TG	T
New York	G	TG	T
Oklahoma	.....	T	T
Minnesota	.....	T or G	T or G
Pennsylvania	G	GT	T
Vermont	.....	G	G
Virginia	.....	T	T
College Entrance Board	.....	GT	T

* In this table, G means general mathematics; T, traditional mathematics; TG, a course between the two types but more like traditional mathematics; and GT, a course between the two but more like general mathematics.

† For the slow group of pupils.

2. The mathematics in the courses investigated show that the work in the ninth grade of the four-year high school is more traditional than the work in the ninth grade of the junior high school. This is in harmony with the results of the textbook analysis. Of the seven city courses obtained for this grade, two are of the traditional type; three are of a type somewhat more like the traditional than the general; and two are somewhat more

like general than traditional mathematics. Of the nine state courses examined, three are of the purely traditional type; three represent a kind of mathematics between the two types; one is of the general type; and two provide for a choice between general and traditional mathematics. The syllabus for the ninth grade by the College Entrance Examination Board is more like general than traditional mathematics.

3. The mathematics courses of study for the tenth grade are much more traditional than for the ninth grade for either type of school organization. Four of the ten city courses of study obtained for the tenth grade are of a purely traditional type; five more are of a type more like traditional than general mathematics; and one is of the general mathematics type. Of the eight state courses, six are of the traditional type, one is of the general type, and one provides for a choice between the two types. There are two tenth grade syllabi by the College Entrance Board. One is of the traditional type, and the other shows the influence of general mathematics by including solid geometry in the year's work.

The writer has made a second examination of the courses of study in order to determine the number of courses which include some work in each of the branches of mathematics. The results are as follows:

Junior High School (Ninth Grade):	
Number of courses examined	13
Number including trigonometry	13
Number including geometry	11
Number including demonstrative geometry	3
Four-Year High School (Ninth Grade):	
Number of courses examined	20
Number including trigonometry	9
Number including geometry	13
Number including demonstrative geometry	0
Senior High School (Tenth Grade):	
Number of courses examined	20
Number including algebra	4
Number including trigonometry	5

This investigation shows that all the courses of study for the junior high school include trigonometry, while Worthington's study (see page 93) showed that only 41 per cent of the junior high schools from which he received replies offered trigonometry. The difference in results is perhaps due to two reasons: First, the number of schools offering trigonometry is probably increasing

each year and, since the present investigation is of a somewhat later date, the per cent would be larger. Second, the courses of study are usually more progressive than the actual practice of teachers in the field. Hence it is not unreasonable to suppose that many teachers omit trigonometry even though it is in the course of study.

The examination of courses of study reveals facts similar to those shown by the study of textbooks. In four-year high school the ninth grade is dominated by the traditional type of mathematics to a far greater extent than in the junior high school. The tenth grade is devoted almost entirely to plane geometry. The results of the examinations of textbooks and of courses of study are in complete accord in respect to these facts.

COURSE OF STUDY BULLETINS

City

1. Course of Study in Mathematics for Junior and Senior High Schools (1926). Fresno Public Schools, Fresno, California.
2. A Course of Study Outline in Mathematics for the Senior High School (1928). Los Angeles School District, Department of Psychology and Educational Research, Los Angeles, California.
3. Mathematics for Junior High Schools (1927). Course of Study Series No. 103. Oakland Public Schools, Oakland, California.

   Mathematics for Senior High Schools (1925). Course of Study Series No. 77. Oakland Public Schools, Oakland, California.
4. Mathematics for Junior High School (1927). Curriculum Bulletin No. 108. San Francisco Public Schools; San Francisco, California.

   Mathematics for High School (1928). Curriculum Bulletin No. 203. San Francisco Public Schools, San Francisco, California.
5. Mathematics for Junior High School (1926). Course of Study Monograph No. 17. Public Schools, Denver, Colorado.

   Mathematics for Senior High School (1927). Course of Study Monograph No. 25. Public Schools, Denver, Colorado.
6. Course of Study—Mathematics, Second Unit (1923). School Document No. 19. Boston Public Schools, Boston, Massachusetts.

   Course of Study—Mathematics, Grade X (1925). School Document No. 4. Boston Public Schools, Boston, Massachusetts.
7. Course of Study in Intermediate School Mathematics, Grades 7B to 9A (1926). Detroit Public Schools, Detroit, Michigan.
8. Mathematics for Grades VII, VIII, and IX (1926). Curriculum Bulletin No. 12. Board of Education, St. Louis, Missouri.

   Mathematics, Algebra-Geometry (1926). Curriculum Bulletin No. 23. Board of Education, St. Louis, Missouri.
9. Course of Study—Mathematics (1924). Board of Education, Bayonne, New Jersey.

10. Secondary Course of Study—Mathematics for Grades VII, VIII, and IX (1925). Public Schools, Trenton, New Jersey.
11. The Course of Study in Mathematics for Grades VII, VIII, and IX (1926). Board of Education, School District of Philadelphia, Philadelphia, Pennsylvania.

Course of Study in Mathematics (1923). Board of Education, School District of Philadelphia, Philadelphia, Pennsylvania.

12. Course of Study in Mathematics (1925). Austin High Schools, Austin, Texas.
13. Mathematics Courses of Study for High Schools (1927–28). Dallas Public Schools, Dallas, Texas.

STATE

1. The Teaching of High School Mathematics (1924). State Department of Public Instruction, Tallahassee, Florida.
2. Course of Study for High Schools, Part III—Mathematics (1923). Department of Education of Kansas, Topeka, Kansas.
3. Notes on the Teaching of High School Mathematics. Maryland High School Bulletin (1925). State Department of Education; Baltimore, Maryland.
4. A Tentative Syllabus in Junior High School Mathematics (1928). The University of the State of New York Press, Albany, New York.

A Tentative Syllabus in Elementary Algebra (1928). The University of the State of New York Press, Albany, New York.

Syllabus in Plane Geometry (1927). The University of the State of New York Press, Albany, New York.

5. Course of Study for Mathematics (1925). State of Oklahoma. Department of Education, Oklahoma City, Oklahoma.
6. Syllabus of the High School Course in Mathematics (1924) Bulletin No. 5. State of Minnesota, Department of Education, St. Paul, Minnesota.
7. Course of Study in Mathematics, Years VII–XII (1927). Bulletin No. 39. Commonwealth of Pennsylvania, Department of Public Instruction, Harrisburg, Pennsylvania.
8. Course of Study in Mathematics (1928). High School Manual and Courses of Study for Vermont, Part X. State Board of Education, Montpelier, Vermont.
9. State Course of Study for the High Schools of Virginia—Mathematics (1928). State Board of Education, Richmond, Virginia.

COLLEGE ENTRANCE EXAMINATION BOARD

Requirements in Elementary Algebra, Advanced Algebra, and Trigonometry (1923). Document No. 107. College Entrance Examination Board, 431 West 117th Street, New York, N. Y.

Requirements in Geometry (1923). Document No. 108. College Entrance Examination Board, 431 West 117th Street, New York, N. Y.

CHAPTER IX

OPINIONS OF HIGH SCHOOL TEACHERS OF MATHEMATICS

Questionnaire sent to teachers

The success or failure of any type of mathematics depends to a large extent upon the attitude of teachers concerning it. In order to ascertain the opinions and preferences of teachers in regard to traditional or general mathematics, the writer sent questionnaires to the mathematics teachers in one hundred forty high schools in nineteen different states. A list of schools in which general mathematics was offered was furnished by reputable publishers. The writer added to the list certain schools in which he knew this type of mathematics was taught. These were paired with other schools in cities or towns of approximately the same size. In all but two cases, the paired schools were in the same state, and in most cases in the same part of the state. This method of equating the schools did not prove entirely satisfactory because some of the schools which were thought to be offering only general mathematics were giving either both types or the traditional type only. The questionnaire was as follows:

Questionnaire

To be filled out by teachers of mathematics

Name of High School .....
Where located .....
Name of Teacher .....
Date .....

There are two plans commonly used for organizing the subject matter of mathematics in the four-year high school. We shall designate these by T and G as follows:

T: Traditional arrangement of mathematics consisting of a year of algebra, then a year of plane geometry or algebra, and so on, with little or no attempt to fuse or unite the different parts of mathematics during the year.

G: General, correlated, or unified mathematics in which the different parts of the subject are fused or united in some way in the year's work. The ninth grade course, for example, will generally be more than one-half algebra, but

with a considerable portion of other parts of mathematics such as intuitive geometry, arithmetic, algebra, trigonometry, and so on, included.

In your experience as a teacher in the high school, have you taught both kinds of courses indicated above?.....If you have taught only one kind, which is it? .....

The questions below are to be answered by those who have taught both kinds of mathematics and by those who have given both kinds sufficient study to have formed definite opinions as to their relative merits.

How many years have you taught type T mathematics in the high school? ..... How many years type G? .....

Which type do you think is the better of the two for high school pupils? .....

Tell briefly why you think it is better. .....
.....
.....
.....

REPLIES RECEIVED

Replies were received from two hundred sixteen teachers in sixty-seven schools. Out of this number, sixty-two favored traditional mathematics, forty-eight favored general mathematics, and thirty expressed certain opinions without definitely stating a preference for either type. The schools which were thought to be offering general mathematics reported slightly better than the others but the difference was not enough to be significant. Of the sixty-seven schools from which reports were received, thirty-three were offering traditional mathematics in the ninth and tenth grades; twenty-one, general mathematics; and thirteen, both types.

TEACHING EXPERIENCE

Table 45 gives the teaching experience of those who prefer traditional mathematics and who furnished sufficient data for such a study. It gives the type of mathematics which was being taught by these people in the ninth and tenth grades at the time the questionnaires were filled out. It also gives the number of years of teaching experience of each teacher and the number of years he has taught each type. Table 46 gives similar information for those who favor general mathematics.

An examination of these two tables shows that, on an average, the teachers of long experience prefer traditional mathematics, Of those who prefer traditional mathematics, 17 per cent have had at least twenty years of experience, 34 per cent at least fifteen years, and 56 per cent at least ten years. Only 5 per cent of those

TABLE 45

TEACHING EXPERIENCE OF TEACHERS WHO PREFER TRADITIONAL MATHEMATICS IN THE NINTH AND TENTH GRADES

Teacher's Number	Type offered in School*	Number of Years of Experience		
		Teaching Traditional Mathematics	Teaching General Mathematics	Total Number of Years
1	T	30	1	31
2	T	29	0	29
3	G1, T2	20	5	25
4	T	20	5	25
5	G	19	6	25
6	G1, T2	20	4	24
7	G1, T2	16	8	24
8	T	20	2	22
9	T	19	2	21
10	G1, T2	12	8	20
11	Both	16	3	19
12	G1, T2	14	4	18
13	G1, T2	10	6	16
14	T	10	5	15
15	T	15	0	15
16	Both	10	5	15
17	T	15	0	15
18	T	15	0	15
19	G1, T2	12	3	15
20	T	9	6	15
21	G1, T2	6	8	14
22	T	11	2	13
23	G	6	7	13
24	G	6	6	12
25	G1, T2	4	8	12
26	G	10	2	12
27	T	10	2	12
28	G1, T2	3	8	11
29	T	7	4	11
30	T	9	2	11
31	T	8	2	10
32	T	6	4	10
33	T	7	3	10
34	T	6	2	8
35	T	7	1	8

*T means two years of traditional mathematics; G two years of general mathematics; and T1, G2 one year of general mathematics followed by a year of traditional mathematics.

TABLE 45 (*Continued*)

Teacher's Number	Type offered in School*	Number of Years of Experience		
		Teaching Traditional Mathematics	Teaching General Mathematics	Total Number of Years
36	Both	4	4	8
37	T	4	4	8
38	G1, T2	6	2	8
39	T	8	0	8
40	T	6	1	7
41	T	4	3	7
42	T	6	1	7
43	G	4	2	6
44	T	4	2	6
45	T	3	3	6
46	T	2	3	5
47	T	3	2	5
48	G	3	2	5
49	T	4	1	5
50	T	3	2	5
51	T	3	1	4
52	T	2	2	4
53	T	3	0	3
54	T	1	2	3
55	G	1	2	3
56	Both	2	1	3
57	T	1	1	2
58	T	2	0	2
59	G	0	1	1
Mean		8.7	3.0	11.7

*T means two years of traditional mathematics; G two years of general mathematics; and T1, G2 one year of general mathematics followed by a year of traditional mathematics.

who prefer general mathematics have had as much as twenty years of teaching experience; 21 per cent as much as fifteen years; and 47 per cent, as much as ten years. In no case has anyone had more than five years longer experience teaching general than traditional mathematics whereas thirty-three have had more than five years longer experience teaching traditional than general mathematics. This fact would certainly influence the preferences somewhat because teachers tend to like the type they know most

TABLE 46

EXPERIENCE OF TEACHERS WHO PREFER GENERAL MATHEMATICS IN THE NINTH AND TENTH GRADES

Teacher's Number	Type offered in School*	Number of Years of Experience		
		Teaching Traditional Mathematics	Teaching General Mathematics	Total Number of Years
60	G	25	4	29
61	G1, T2	22	3	25
62	G	16	3	19
63	G1, T2	14	4	18
64	G	10	7	17
65	G1, T2	11	5	16
66	G	8	8	16
67	Both	6	9	15
68	T	10	5	15
69	G	8	6	14
70	G	6	7	13
71	Both	9	3	12
72	G	6	6	12
73	Both	7	5	12
74	T	8	4	12
75	G	8	3	11
76	T	8	2	10
77	G1, T2	5	5	10
78	G1, T2	4	6	10
79	T	7	3	10
80	G	2	7	9
81	T	5	3	8
82	G	4	4	8
83	G1, T2	6	2	8
84	T	8	0	8
85	G	3	5	8
86	T	8	0	8
87	G	3	5	8
88	G1, T2	6	1	7
89	G	6	1	7
90	T	4	3	7
91	G1, T2	4	3	7
92	G1, T2	3	4	7
93	T	7	0	7
94	T	6	0	6

* See note to Table 45.

TABLE 46 (*Continued*)

Teacher's Number	Type offered in School*	Number of Years of Experience		
		Teaching Traditional Mathematics	Teaching General Mathematics	Total Number of Years
95	G	4	2	6
96	G	0	5	5
97	G	0	5	5
98	G	4	1	5
99	T	2	3	5
100	T	5	0	5
101	G	2	2	4
102	G	1	1	2
Mean		6.8	3.6	10.4

* See note to Table 45.

about. Most of those who have taught general mathematics the greater number of years probably were taught traditional mathematics in the elementary grades and the high school so that in almost every case the total experience with traditional mathematics was at least as great as with general mathematics. The fifty-nine teachers who prefer traditional mathematics have had an average of 11.7 years of teaching experience, 7.8 years of which have been in traditional mathematics and 3.0 years in general mathematics. The forty-three teachers who prefer general mathematics have had an average of 10.4 years' teaching experience, 6.8 years of which have been in traditional mathematics and 3.6 years in general mathematics. This shows that both groups have had far greater teaching experience in traditional than in general mathematics. It would be interesting to know the opinions of the teachers if they had had the same amount of experience with both types from the beginning of their junior high school education.

REPLIES FROM GRADUATE STUDENTS

Copies of the same questionnaire were given during the summer of 1928 to more than two hundred graduate students of Teachers College, Columbia University. All these students were enrolled

in classes in the teaching of mathematics and nearly all of them had been teaching mathematics during the past year in either a junior or a senior high school.

The reactions of the students were quite different from those of the teachers in the field to which reference has already been made. Of the one hundred ninety-five who returned the questionnaire, six preferred traditional mathematics and seventy-five favored general mathematics. The others did not express a preference. Of the eighty-one who expressed a preference, eighteen had taught at least some classes in general mathematics during the preceding year, fifty-three had taught only traditional mathematics, and ten either did not teach or did not report the type taught. At least two factors may have influenced the decisions of this group of teachers. First, all of them were having work with Professor W. D. Reeve who advocates general mathematics. Most of them, however, had been in his classes only five weeks, and no attempt was made to influence their reactions one way or another in respect to this questionnaire. Everyone was asked to record exactly what he thought. Second, these teachers were for the most part progressive, as is indicated by the fact that they were graduate students seeking more education. To be sure, many of the high school teachers in the field to whom questionnaires were sent were just as progressive and their opinions are as valuable but they did not represent so select a group as the Teachers College students did.

REASONS FOR PREFERENCES

Reasons for preferring the particular type of mathematics indicated were given by fifty-nine of the sixty-two teachers who preferred traditional mathematics, by forty-eight or all of those who preferred general mathematics, by thirty who did not express a preference, and by eighty-three students of Teachers College, Columbia University, all but six of whom preferred general mathematics. Almost every teacher expressed his reasons in a different way, and it was consequently extremely difficult to tabulate them. After several attempts, the writer succeeded in grouping them under a number of somewhat arbitrary heads. The analysis is given in Table 47. In many cases, the statement of the reasons had to be changed somewhat in the grouping and sometimes the meaning may have been altered a little. It was thought best to

TABLE 47

REASONS GIVEN FOR PREFERRING A PARTICULAR TYPE OF MATHEMATICS, AND NUMBER OF TEACHERS GIVING EACH REASON

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				KNOWLEDGE OF MATHEMATICS
18*		4	1	T develops a better knowledge of mathematics.
	9		6	G develops a better knowledge of mathematics.
				Partial analysis:
5			1	T gives a better knowledge of the fundamentals.
2	1	2		T gives a more thorough knowledge of mathematics.
16				G gives only a smattering knowledge of mathematics.
1		2		T gives a better working knowledge of algebra.
1		1		T gives a more adequate knowledge of algebra at the end of the ninth grade.
1		1		T gives a better knowledge of geometry.
	3		4	G gives a better knowledge of the fundamentals.
	2			G gives a more thorough knowledge of mathematics.
			2	G develops the function concept more readily.
	2			G keeps algebra fresher in mind
		2		G fixes geometry firmer in mind.
1				G develops as good a knowledge of algebra and geometry by the end of two years.

* This number does not represent the total in the partial analysis. It indicates the number of favorable opinions as shown by the individual replies. In this way, repetitions and omissions in the partial analysis have been avoided. The same statement may be made of various other numbers throughout the table.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				CONCEPTION OF MATHEMATICS
7			1	T gives a better or broader conception of mathematics.
2	11	5	39	G gives a better or broader conception of mathematics.
				Partial analysis:
5			1	T gives a better understanding of mathematics.
2				T gives a better idea of the laws of mathematics.
2	6	4	25	G gives a broader conception of the field of mathematics.
1	3	1	6	G is a better exploratory course for determining the likings and aptitudes of the pupils.
			4	G gives a better appreciation of mathematics.
			1	G gives more insight into the beauty of mathematics.
				MATHEMATICAL ABILITY
16		2		T develops more real mathematical ability.
	10	1	11	G develops more real mathematical ability.
				Partial analysis:
8	1	2		T provides for more adequate drill.
4				T develops a better mastery of mathematics.
4				T develops better thinking and reasoning ability.
1				T develops better ability to analyze and solve problems.
	1		5	G provides for more adequate drill and review.
	1		1	G develops better mastery of mathematics.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
	5		3	G develops better thinking and reasoning ability.
	4		1	G develops better ability to analyze and solve problems.
1	1	1		G strengthens the arithmetical ability.
				PRACTICAL VALUE AND APPLICATIONS
0				T has greater practical value.
	10	3	22	G has greater practical value.
				Partial analysis:
	3		7	The subject matter of G is more closely related to the needs and experiences of the pupils.
	3		6	G presents a type of mathematics more like that used in everyday life.
			1	G enables pupils to see more clearly the need of mathematics.
				INTEREST
			4	T makes a greater appeal to the interest of the pupils.
	17	1	30	G makes a greater appeal to the interest of the pupils.
				Partial analysis:
	6		2	G creates greater interest because of the variety.
	2		1	G presents more interesting subject matter.
	2			G creates more interest in mathematics.
	1			G creates an incentive to continue mathematics.
			1	G makes the reviews more interesting.
	3			G prevents discouragement due to one subject being more difficult than another.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				CORRELATION
6	17	1	11	The subject matter should be correlated or unified to a certain extent.
5		1		Correlation or unification of the subject matter should not be attempted.
				Partial analysis:
6				There are only a few places where true correlation takes place.
1				G takes away teacher initiative in fusing the subjects.
1		1		Many of the correlations in G are forced.
2	2	1	1	Demonstrative geometry does not correlate readily and should be kept distinct.
1				There can be enough fusion in T courses.
	1			G is better if it does not fuse the different branches.
1				Correlations should come after three or four years of T.
	5		7	G correlates (fuses) the different branches better.
	6		4	G gives a view of the unity of mathematics.
3	3	1	2	Intuitive geometry enriches algebra.
2	2		1	G makes use of algebraic methods in the study of demonstrative geometry.
1	4		2	One branch of mathematics helps another.
			1	Fusion makes new material more acceptable.
	1			Correlations can be made easily and naturally.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
	1			Correlation gives more mathematical power.
			2	Correlation saves time.
	1			All modern movements for the improvement of mathematics correlate something.
				ORGANIZATION AND GRADING OF COURSES
21		1		T is better organized and graded.
1	7		10	G is better organized and graded.
				Partial analysis:
2				T develops continuously.
4				G crowds too much into a year.
4		1		G is not yet well organized.
10				G confuses the pupils by too frequent changes of the subject.
1				Algebra is better graded to pupils' progress.
3				T is more connected.
1				Better results are secured by concentrating on one thing at a time.
1				T is logically correct.
1	1		1	T is offered in most schools and the transfer from one school to another is easier.
2	1			G is psychologically correct.
1			1	G is logically correct.
	3		3	G presents the subject matter in a more natural sequence.
1	1		2	G eliminates unnecessary material.
			1	G provides for the introduction of subject matter in the order of necessity.
			1	G provides for the introduction of relevant topics.
	2		1	G presents easier material first and the more difficult later.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				TEXTBOOK DIFFICULTIES
8	4	2	1	Dissatisfaction expressed with G textbooks.
				Partial analysis:
2	2	1	1	G textbooks are not desirable.
1				G textbooks are too easy.
2		1		Textbooks we have used in type G are not good.
1				G textbooks are poorly organized.
		1		G textbooks are theoretically sound.
2		1		T textbooks are better than G textbooks.
1				T textbook used is a mixture of unrelated material.
				RESULTS EXPERIENCED
2				Experience has proved that results are better with type T.
1			1	Never knew a trial of G to be successful.
1				In the experiments I have watched, the pupils were not prepared in any branch of mathematics.
	1			Believe in G but abandoned it because the older books were not satisfactory.
			1	Believe in G but do not think I was successful.
	1			Used G in evening classes with excellent results.
		1		Achieved rather satisfactory results with G.
	2			Disliked G very much at first but now the longer I teach it the better I like it.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				TYPE OF TRAINING DESIRED
7	1	8	1	T is better preparation for college.
	3		1	G is better preparation for college.
11		7		T is better preparation for higher mathematics.
	1		5	G is better preparation for higher mathematics.
		2		T is better for technical school pupils.
2	3	2	1	G is better for those who take only one year of mathematics.
1	2	5	2	G is better for those who do not go to college.
1		1		G is better for vocational pupils.
				TYPE OF PUPILS
1		2		T is better for bright pupils.
	1		1	G is better for bright pupils.
2		2		T is better for average pupils.
4	1	2		T is better for slow pupils.
		1	1	G is better for slow pupils.
2			1	T is easier.
2	3	2	7	G is easier.
				OTHER COURSES OR WORK GIVEN
1				The type that is taught in the junior high schools is better.
1	1			G is good for junior high school pupils if followed by T in the senior high school.
1		1		G is better for junior high school pupils and T for senior high school pupils.
		1		G would be better if demonstrative geometry were dropped from the course.
	1			G is good if intuitive geometry is introduced in the grades.

TABLE 47 (*Continued*)

No. of Teachers who prefer:			No. of Teachers College students	Reason
Traditional Mathematics	General Mathematics	Preference not stated		
				MISCELLANEOUS
1		1		Prefer G to old type of algebra and geometry but believe new algebras and geometries are better.
	2	1		Intuitive geometry in the grades makes demonstrative geometry easier and less formal.
			2	G has more meaning to the pupils.
	1			G takes account of all types of pupils.
	1			Three years of G is as good as three and one-half years of T.
		1	2	G is better if teachers are well prepared.
	1			Impossible to train teachers now in service to teach G mathematics.

do this for the sake of making the table as simple as possible. Every pertinent reason is recorded somewhere in the list. Because of the necessity of grouping and condensing, the table is not wholly objective. The views of the writer necessarily entered into determining how the grouping should be done. The reasons are grouped under thirteen heads as follows: (1) knowledge of mathematics, (2) conception of mathematics, (3) mathematical ability, (4) practical value and applications, (5) interest, (6) correlation, (7) organization and grading of courses, (8) textbook difficulties, (9) results experienced, (10) type of training desired, (11) type of pupils, (12) other courses or work given, and (13) miscellaneous. Since this classification is arbitrary, it is not probable that anyone else would have made exactly the same grouping. Nevertheless, it reveals certain facts concerning the various beliefs which teachers have about the two types of mathematics.

OPINIONS OF TEACHERS AS REVEALED BY THE TABLE

Table 47 reveals the following facts concerning the opinions of teachers whose opinions are tabulated:

1. There is a somewhat stronger belief in the value of traditional mathematics for giving the pupils a knowledge of pure mathematics. The chief criticism of general mathematics, in this respect, is that it gives a smattering knowledge of several subjects and not an adequate knowledge of any one of them.

2. A much stronger claim is made for general mathematics concerning its value in giving a broader and better conception of what mathematics really is. Perhaps the strongest claim made for general mathematics is that it gives a broader conception of the field of mathematics and that it is a better exploratory course for revealing the likings and aptitudes of the pupils.

3. There is not much difference in the relative number who favor each of the two types in the belief that it will develop better mathematical ability. A slight preference would seem to be in favor of traditional mathematics. The advocates of traditional mathematics lay considerable stress upon its provisions for adequate drill, while those of general mathematics divide their emphasis among adequate drill and review, better ability to solve problems, and better thinking.

4. Many of those who favor general mathematics make claims as to the practical value of this type. Those who favor traditional mathematics do not even suggest its practical value.

5. One strong claim of the general mathematics advocates is that it creates greater interest. Very little is said by anyone about the interest aroused by traditional mathematics.

6. Most teachers who express any opinion concerning correlation favor at least some correlation of mathematical subjects.

7. General mathematics is criticized for being poorly organized and graded. Its most severe condemnation in this respect is based upon the claim that it confuses the pupils by too frequent changes of the subject matter. This is perhaps due in part to the fact that not nearly so many teachers have been working to perfect the organization of the material of general mathematics as have done so in traditional mathematics.

8. There is some dissatisfaction with general mathematics textbooks. This is to be expected for at least three reasons. First, there are several times as many of the so-called traditional

type textbooks as of the general type and hence it is easier to find a traditional type book suiting one's particular fancy. Second, the writers of the general type textbooks have not been able to profit to as great an extent by the mistakes and successes of others because of the fewer number of books of this type. Third, the general mathematics books are so different from the type which most teachers have studied that it is rather natural to think that they are not well written.

9. The reports of results of experiments indicated by teachers' opinions are too meager to be of any value.

10. Traditional mathematics is the favorite for preparation for college or for higher mathematics in high school. This is quite natural since nearly all the college mathematics courses and most of the third and fourth year high school mathematics courses are of the traditional type. Teachers would naturally expect better results if the same type were taught in the ninth and tenth grades as is taught in the eleventh and twelfth grades and in college. Such a conclusion is not warranted, however, in view of the experimental study of the marks made by freshman students at the University of Minnesota. (See page 41.)

11. More teachers seem to think that general mathematics is easier than traditional mathematics. General mathematics is favored by a greater number of teachers for the slow pupils than for the bright pupils.

The list of facts just given is far from being exhaustive, but it contains what seem to the writer to be the most important ones. A study of the table will reveal many other conclusions some of which may be of particular interest to certain readers. This will be left for individual study by those interested.

CERTAIN OUTSTANDING OPINIONS

Some of the opinions were particularly well expressed but suffered by being broken up into single units for Table 47. A few of these opinions will be given in full. The first two quotations are from teachers in the St. Louis schools. The St. Louis course of study provides for general mathematics in the ninth grade and a parallel arrangement of algebra and geometry in the tenth and eleventh grades. (See page 102.) From remarks quoted on the following pages, it is apparent that an algebra course is also being offered in the ninth grade.

A St. Louis high school teacher who expressed a preference for traditional mathematics, commented as follows:

In my brief experience with general mathematics, I achieved rather satisfactory results, but could not present enough algebra to have the pupils able to carry on the advanced work in algebra. The course as presented in the St. Louis schools was intended only for those who expected to take but one year in mathematics. We offer both general mathematics and algebra in the ninth grade, and urge those who expect to take more than one year of mathematics to choose algebra during the first year.

Another St. Louis teacher, who favors general mathematics, attempted to evaluate the results observed in that city:

It is difficult to evaluate general mathematics from experience with it in the St. Louis high schools. It has never been given a fair chance. It has been taught with (1) a poor book put into the hands of the children, a use for which its authors did not intend it, (2) no uniform plan of procedure, (3) teachers who took it under protest and as an odd class, (4) later term courses planned to follow a course in traditional algebra instead of fitting on to a prerequisite general mathematics course and taught to students from both preliminary courses at the same time, and (5) classes in general limited to C type intelligence, vocational courses, or avowed disinterest in mathematics.

The emphasis placed upon drill by the advocates of traditional mathematics was very well expressed by a San Francisco teacher:

It is a well-known fact psychologically that a study or subject is learned by continuous repetition and drill. The wise teacher in the traditional method will give continuous practice but nevertheless occasionally show the interrelations with other branches of mathematics, so that algebra, geometry, and trigonometry will be regarded as stepping stones rather than isolated rocks in the stream of mathematics.

General mathematics, like general science or general culture, is conducive to only a very superficial understanding of the subject involved. . . . It seems to me that the traditional method of hard work leads to thorough understanding and that generalities lead to inane glibness and superficiality.

The teacher just quoted overlooked one important condition imposed upon the psychological law of repetition; namely, "other things being equal." Drill which becomes a monotonous grind, is not likely to produce valuable results. A course in pure algebra, for example, is likely to become tedious because of the apparent sameness of the material. At present in many of our mathematics courses, too much time is spent in drill on useless types of examples for apparently no other purpose than to enable pupils to work more examples of the same type. General mathematics provides abundant opportunities for repetition of worth-while

subject matter. It does neglect, however, material which cannot be used in a new setting. This seems to the writer to be a strong argument in favor of general mathematics because the more usable a subject becomes the more valuable it will be. Mere drill which does not result in learning that which is worth while is often a waste of time.

A Wisconsin teacher expressed an adverse criticism of the general mathematics textbooks:

> Perhaps we have become unduly prejudiced against general mathematics because of the textbooks we have used. There is lack of simple drill material, and too much material requiring long and involved procedures. The material is poorly presented. . . . While we will not be permitted to change textbooks for next year, we have practically decided to teach algebra straight through the first year and plane geometry the second, using outlines and supplementary material.

In fairness to the author of the textbooks to which reference has just been made, it should be stated that the books used represented an early edition. A much later edition has appeared which has eliminated most of the faults to which this teacher referred.

A teacher in one of the better high schools in Iowa, who stated that she favored traditional mathematics, said:

> I think general mathematics has a permanent place for grades seven and eight. I prefer general mathematics for grades nine and ten to the old-type algebra and geometry which we had before the introduction of general mathematics. The modern algebra and modern geometry I believe superior to general mathematics. Beyond the tenth grade I have had no experience with general mathematics.
>
> The general mathematics movement for grade nine introduced material suitable for the grade in which it was taught, omitted nonessentials and used much better psychology of learning in introducing new topics. It failed to give sufficient drill on any one topic and it introduced geometric and other material which advanced the algebra but very little but did detract much from it because of an almost complete and lengthy shift of attention. The new algebra embodies the good without the bad of the general mathematics.
>
> Geometry suffers most from attempts at fusion. It has always received much criticism because of the number of failures. This is not helped by lessening the amount of material and introducing other material at least as difficult with less time left to spend on the formal type of reasoning in plane geometry.
>
> The new type of geometry does not have as many theorems as the old time geometry but multitudes of very simple originals in the beginning of the course and throughout the year. . . . In my opinion we must have an entire year thus spent on plane geometry alone if the ideals of geometry are to be attained.

The preceding quotation expresses a view advanced by a few teachers in opposition to attempts to fuse demonstrative geometry with the other branches of mathematics. It also reveals the fact that if general mathematics is to secure a permanent place in the schools of America, it is necessary to keep the textbooks as up to date and modern as the books on algebra and plane geometry. It is quite probable that this teacher, however, is better acquainted with the general mathematics books published several years ago than with those of most recent publication.

From one of the high schools in Minnesota, a teacher sent a statement of her reason for discontinuing the general course. She said:

> One reason we returned to the traditional course in full was that we had put the general into only the ninth grade, keeping the plane geometry in the tenth grade, and higher algebra and solid geometry in the eleventh and twelfth grades. This caused more confusion, of course, than had we had the full course in general mathematics. We considered it unwise to put the general course into the tenth grade, as the field was too great for one year. We have not regretted returning to algebra. The new algebra texts are a great improvement over texts written previous to 1920, and do fuse different parts of mathematics, while the older ones do not.

The present writer is of the opinion that this teacher did not return to "the traditional course in full." If a textbook was used which does "fuse the different parts of mathematics," the author must have caught some of the spirit of general mathematics. This quotation shows what several other expressions of opinion do, that teachers may think they are teaching traditional mathematics because they use a book called "algebra," when in reality it may be a very good type of general mathematics.

A Minneapolis teacher compared our textbooks with European textbooks as follows:

> In fact, when the textbook writers of this country have mastered the same technique that the textbook writers of England, France, and Germany have done, it [general mathematics] will, no doubt, be the only method. At present few of our texts are thoroughly enough committed to type G to make a general course based on these texts satisfactory.

An Iowa teacher cited her experience with general mathematics:

> After a year or perhaps two years of teaching general mathematics I still felt it to be of no value, but now the longer I teach it the more I prefer it to algebra and geometry. This year I have two classes in G and two in T mathematics. Of the four groups, I prefer the two G groups on the whole. Of

course there are individual differences but the G groups have algebra fresher in mind that the T groups and accomplish better problem work.

The teacher just quoted has taught type T for six years and type G for nine years. A Michigan teacher who has taught type T for four years and type G for six years expressed a similar view:

I disliked the "G" type very much when I started to teach it, but I am now convinced that it gives the pupil a better understanding of mathematics. The "T" type of teaching algebra is often mechanical and not very enlightening even to the brighter pupils.

If these two quotations are typical of the experiences of the majority of teachers, they are very significant. They would indicate that many teachers may have abandoned general mathematics before they had taught it long enough really to appreciate this type of mathematics. Most of them have been taught the T type in the elementary grades, in high school, and in college so consistently that it takes considerable time for them to get as thoroughly acquainted with the G type. It would therefore seem that the experiences of these two teachers may be typical of a large majority of the teachers of mathematics.

Another expression of opinion which warned against hasty conclusions came from a Minneapolis teacher:

We have not given type G a fair chance. We have not had type G in both ninth and tenth grades more than three or four years, if that long. I do not approve of type G in the ninth grade and type T in the tenth grade as we have it now, unless a third year of mathematics is introduced in which nothing but algebra is taught.

The objection just expressed to general mathematics in the ninth grade and plane geometry in the tenth grade is in conformity with the results of Professor Bussey's study. (See page 40.) He found that a year of each type was not a good preparation for college mathematics. Too much geometry is given apparently at the expense of a more thorough training in algebra.

Many advocates of general mathematics think that mathematics is greater than merely a collection of its parts. This opinion was expressed by a teacher in Illinois:

By the end of a year or more of type "G" a student actually realizes that mathematics is not just algebra alone or geometry alone, but that a thorough knowledge of one depends quite a bit upon the other. I feel that three years of type "G" are better than three and one-half years of type "T." It is better for fast or slow classes.

One of the most frequent arguments in favor of general mathematics is its practical value. Claim is made that it is more closely related to the life of an individual. Another teacher in Minneapolis expressed this view:

> General mathematics has in it the type of training to develop a well-ordered life. Thus we have the opportunity to teach a better type of quantitative thinking which is similar to the requirements of everyday life. Also, it is the mathematics that individuals should use in everyday life.

In recent years, there has been considerable agitation for a general mathematics course in which the emphasis is not placed upon correlation. In fact, the tendency at present is away from the highly correlated course which was advocated several years ago. This view of general mathematics was expressed by a St. Louis teacher:

> I think a combination course with units of number and form work in each term is the better—*not a fused course.* It distributes the material over a greater period of time. It enables the student to study the easiest portions of the subject early in his school career, and to postpone the more difficult portions. By distributing the work over a long period of time, the students grow into a knowledge of mathematics naturally. The correlations can be made naturally and easily.

One more quotation of opinions given will be included in this chapter. It is from a foreign student at Teachers College, Columbia University:

> I taught only one year to the tenth and eleventh grades in high school. The course of study planned by the school was a year of algebra and a year of plane geometry.
>
> I had never heard of general or unified mathematics. I had been taught the above subjects separately as in the traditional way.
>
> When I began to teach algebra to the tenth grade and geometry to the eleventh grade, I did not see how I could do it without mixing in a little of each with the other. It worked out beautifully and made my work easier.
>
> To my surprise, when I came to the United States, I found out that what I had done was being introduced into American high schools.

These quotations show that both the advocates of traditional mathematics and the advocates of general mathematics are trying to break down the separation between the branches of the subject. The real difference between them is as to how far one would wish to go in the process. It is apparent that some teachers would call a certain progressive type of mathematics traditional and others would call it general. This does not mean that there is not a

difference in the beliefs of the two groups as to what constitutes general mathematics, but rather that there is a great deal of overlapping and that the replies as to preference do not always state exactly the type preferred. Most of the replies show very definitely the influence of the general mathematics movement in breaking up the strict separation of mathematics into different branches and establishing a greater unity in this particular field of learning.

CHAPTER X

OPINIONS OF SPECIALISTS IN METHODS OF TEACHING MATHEMATICS

In the preceding chapter, a report was made of the opinions of high school teachers of mathematics with respect to whether general or traditional mathematics is the better type for high school pupils. Some of these teachers have had wide experience and are of exceptional ability. In general, however, they represent an unselected group, some being better and others worse teachers than the average. The viewpoints of the teachers in any subject matter field are quite valuable, but most of the progress which is made is first suggested by opinions expressed by specialists in the methods of teaching the subject. It is the purpose of this chapter to record the opinions of some of the leading specialists in the teaching of mathematics and to list published articles relating to the merits of the two types.

HOW THE OPINIONS WERE SECURED

Perhaps the best way to obtain the opinions of any group of people is to consult the articles and books which they have published. The writer has made a somewhat extensive investigation of the publications issued since 1900 which relate to the relative merits of traditional and general mathematics. He found eighty-six different publications by sixty-two different authors which either stated the author's views in regard to the two types of mathematics or gave some inference as to what he thought. In addition, a number of reports of experiments were found which reflected opinions as well as experimental results. References are made to these experiments in Chapters III and IV. Part of the opinions were expressed by specialists in the teaching of mathematics, and most of the remainder were by high school teachers who had apparently given considerable thought to the methods of teaching mathematics. The publications referred to above are listed in an annotated bibliography at the end of this chapter.

Before discussing the results of a careful analysis of the bibliography, a few quotations from some of the leading specialists in the teaching of mathematics will be given. Some of the best-known experts have not expressed their opinions of the two types of mathematics in any published article, so far as the writer was able to discover. He therefore sent inquiries to a number of them, asking for a statement of their opinions of the relative merits of general and of traditional mathematics.

OPINIONS SECURED BY INQUIRIES

The replies obtained from this solicitation of opinions will be given first. A request for the information desired for the report given in Chapter IX was sent to the New Mexico Normal University, East Las Vegas, New Mexico. The following reply from the dean, T. G. Rodgers, was received:

> Your questionnaire was received and I regret to state that we have done nothing in this line for several years and so are not in a position to fill out the paper.
>
> In 1914, I began a series of experiments on the reorganization of secondary mathematics and carried on this for six years. The work centered about forms of general mathematics. . . . During this time I felt that general mathematics in some form is decidedly the best for high school work and those who assisted me agreed with me, but my duties became so heavy that I could not continue direct supervision of the work and my assistants, who were new, gradually gave up the matter. My experience has shown me that both students and faculty, unless you have a superior faculty, prefer the old line of mathematics. It requires more thought on the part of both teacher and pupils to work general mathematics and that is one of its real values but the line of least resistance is to go on with the old, improved as much as possible.

This reply of Dean Rodgers is quite different from the replies of many high school teachers who can see nothing in general mathematics except a smattering of this and that without the necessity of much effort on the part of the pupils. A report of Rodgers' experiments may be found on pages 245–51 of the National Committee's Report.[1]

Professor H. C. Christofferson of the School of Education, Miami University, Oxford, Ohio, has had considerable experience with general mathematics. He observed the teaching of it in the University High School of the University of Minnesota, secured

[1] *The Reorganization of Mathematics in Secondary Education.* A Report by the National Committee on Mathematical Requirements under the Auspices of the Mathematical Association of America, Inc. (1923).

first-hand information of its success in the high schools of that state, and made an extensive study of the teaching of mathematics both as a college professor and as a graduate student. His opinion was expressed as follows:

> The arrangement of mathematics known as "General Mathematics" has possibilities which make it distinctly superior to the traditional separation of high school mathematics into Arithmetic, Algebra, Geometry, and Trigonometry. While it certainly seems unfortunate to forbid the geometry teacher the use of algebraic symbolism or the algebra teacher the use of geometric figures to illustrate certain algebraic processes, it is probably equally vicious to try to force the teacher and student to mix these courses artificially and to assume a common purpose where it does not exist. Either extreme is undesirable.
>
> To consolidate all algebra teaching into one year and all geometry into another rather than to have some intermingling of the courses violates an important law of the psychology of learning, namely, the distribution of practice periods. Those of us who use algebra in our geometry courses and occasionally devote a whole period to practice on algebraic problems realize how quickly children forget and how easily they relearn and more completely master an algebraic technique.
>
> General Mathematics uses better psychology of learning, does not handicap or limit the teacher by forcing him to confine himself to a single field of mathematics, and is a much more real and natural organization. These and other virtues will eventually demonstrate the superiority of "General Mathematics" over the traditional organization.

The above statement is an excellent expression of some of the reasons for teaching general mathematics. One of the leading specialists in the teaching of high school mathematics in the state of New York is William Betz, Vice Principal and Director of Mathematics, East High School, Rochester, New York. He has been a leader of reform in the teaching of junior and senior high school mathematics in that state for a number of years. His reply was as follows:

> A perfect solution of the question of *general* mathematics is, of course, impossible within the customary time limits. The crux of the whole problem is not merely the new content, but especially the time schedule. It is absurd to talk merely about the ninth and tenth years. General mathematics should begin in the kindergarten and should be continued in the university. At present, it has no chance whatever, so long as we insist upon the absurd notion that it is a question merely of the upper high school.

There is considerable strength in Betz's argument that general mathematics should extend through all the mathematics work. It does not seem sensible to say that there is a time in a pupil's growing knowledge of mathematics when one branch should be

related to another and that at other times they should be kept separate. In fact, most of the arguments in favor of general mathematics do not apply to any particular time or place in the study of the subject.

There are two specialists in mathematics who devoted practically their entire time for several months to the preparation of the National Committee's Report. One of them is Professor J. A. Foberg, formerly of the State Department of Public Instruction of Pennsylvania, now head of the mathematics department of the State Teachers College in California, Pennsylvania. He replied to the question in regard to the merits of the two types of mathematics as follows:

> General mathematics, as a type of organization of mathematics material of instruction, is superior to the traditional compartmentalized organization. I look upon the definition of Algebra I in the College Entrance Examination Board's Manual as a good type of general mathematics.

The definition of elementary algebra (Algebra I) in the manual is stated in these words:

> In conformity with the general principles above laid down, a less extensive treatment of certain purely algebraic topics than has hitherto been the practice in elementary algebra is herewith specified, in order that time may be saved for the introduction of intuitive geometry and numerical trigonometry, as well as for better emphasis on the great basal principles of algebra. The examinations are expected to be more searching with respect to the parts of algebra that will be used by the pupil in his later work, and particularly with respect to his ability to solve applied problems, to handle formulas, to interpret graphs, and to solve the types of equations which he will subsequently need to use.

One of the best-known specialists in the teaching of mathematics in the state of Oklahoma is Professor J. O. Hassler of the University of Oklahoma. He is also a co-author of a series of junior high school textbooks. In reply to the question about the relative merits of general and traditional mathematics he said:

> I believe in fusion where the nature of the subjects is such that they ought to go together, e.g., Proportion (algebra), Similarity (geometry), and Solution of Triangles (trigonometry); or formulas of Mensuration (algebra) and their proofs (geometry). I do not believe in forced fusion of unrelated subject matter.

The views of Professor Hassler are in harmony with the present trend in the correlation of mathematical subjects. Whenever correlation of the branches is brought about, it is claimed that

it should be natural. Another author of a series of junior high school textbooks is H. C. Barber, Exeter, New Hampshire. Besides being an author and a teacher of wide experience, he is president of the National Council of Teachers of Mathematics. He replied to the question as follows:

> In studying any topic in mathematics, it seems to me stupid not to make use of all available methods which are helpful in producing the results sought or in illuminating the field of mathematics, even though these methods may be classified under other mathematical heads than the particular one under discussion.
>
> However, I do not want to be put on record as favoring general mathematics as some other person at some other time might choose to define the term.

The view here expressed is gaining in popularity every year. The mutual helpfulness of the subjects has been one of the strong arguments in favor of general mathematics. There have been so many different interpretations and so much misunderstanding of general mathematics that the author exercised a wise precaution in making a reservation as to how the term is to be defined if he is to be put on record as favoring this type of mathematics.

Another teacher, Professor G. W. Myers, of the University of Chicago, did more in the early stages to develop correlated (general) mathematics in this country and to cause it to be introduced into the schools than anyone else. He has had a great deal of experience with mathematics of the two types both from the high school and from the college viewpoint. He taught mathematics in the University High School for many years at the time when correlated mathematics was in the formulative period. For a number of years he has been teaching in the college department alone. His reply to the question was as follows:

> Of course, it depends on the type of general mathematics. Much that calls itself general mathematics is of small value. General mathematics by educational writers who know little mathematics and dislike the subject of mathematics itself is worse than wasted time. I care little for the mathematical guidance of any author who does not believe in the high educational worth of mathematics treated either as isolated special courses and topics or as unified or general or fusion material. Away with those who would "reform" mathematics out of the curriculum.

SOME OPINIONS ALREADY PUBLISHED

A few quotations from the publications of some of the leading educators and specialists in the teaching of mathematics will

now be considered. Professor C. H. Judd of the University of Chicago is a general educator who is a true friend of mathematics. He is so well known and is such an ardent advocate of mathematics that his opinion should carry considerable weight with those who are trying to improve the subject. On pages 98 and 99 in his book entitled *Psychology of Secondary Education*, he said in regard to combination mathematics:

> The experience of teachers of mathematics has led a number of leaders in this field to the conclusion that the specialization characteristic of mature algebra and mature geometry is not appropriate to the early stages of the study of these sciences. There has accordingly arisen of late a vigorous movement in the schools to break down the traditional separation between algebra and geometry and to teach algebraic formulas and geometric formulas together for the purpose of giving the pupil training in the related though somewhat different types of experience. This combination of the mathematical sciences makes it possible to use one sphere of experience as an opportunity to apply the principles arrived at in the other sphere. It also makes possible in the early stages concentration on the more elementary phases of both sciences. Combination mathematics, as the fusion of algebra and geometry is called, explains algebraic abstraction to pupils by means of figures and formulas from geometry, and vice versa. The result seems to be that pupils more readily understand both types of reasoning and gain more complete mastery of the general principles involved in both sciences. Furthermore, pupils trained in combination mathematics exhibit a more general ability to carry over the types of thinking acquired in such a course to the sciences of physics and chemistry and to the practical problems of engineering.

Professor T. P. Nunn of the London Day Training College, an English specialist in the teaching of mathematics, will be quoted next because of the great influence he is having upon the secondary mathematics in our country. On pages 19 and 20 of his book, *The Teaching of Algebra*, he said:

> The foregoing principles point directly to two important modifications in present teaching practice. The first concerns the position of trigonometry, the second that of the differential and integral calculus. At present it is usual to teach both of these as "subjects" distinct from algebra and distinct from one another. The reason is easily found. The professional mathematician, interested chiefly in perfecting the technique of his subject, finds it natural as well as most effective to take a special group of allied methods or allied problems and to develop them as far as he can without concerning himself too greatly about the practical value of his work. Now this systematic exploration of special parts of mathematics is, no doubt, of vital importance for the continued growth of the science. It does not, however, by any means follow that the branches of mathematics should be presented to beginners with the formal elaboration which is the inevitable mark of their treatment as separate subjects. . . .

If we admit that the custom of teaching trigonometry and the calculus as distinct subjects has no defensible foundation and is responsible for very unsatisfactory consequences, we shall be prepared to inquire what position is assigned to them by the principles laid down in this chapter. Taking first the case of trigonometry, it becomes at once evident that the pupil's acquaintance with the tangent, sine and cosine should begin in the region where arithmetic marches with elementary geometry. As soon as the symbols of the trigonometric ratios are recognized as capable of entering into formulae and of being manipulated they should be regarded as belonging to the vocabulary of algebra. . . . Both algebra and trigonometry would gain by fusion; the former through an added variety and richness in the illustrations of its main themes; the latter by the removal of the excessive formalism which at present obscures its value and interest for the beginner. . . .

In arguing for the assimilation of trigonometry in the algebra course we are arguing against an artificial separation between problems which must in any case be attacked in a similar spirit and by similar weapons.

An examination of this quotation from Professor Nunn's book shows that he was arguing for a type of general mathematics. He said that acquaintance with the trigonometric functions "should begin in the region where arithmetic marches with elementary geometry." Later on he said that "both algebra and trigonometry would gain by fusion." Thus he advocated the use of arithmetic, algebra, geometry, and trigonometry, all in one course. He opposed an unnatural separation of the subjects. Some American authors and teachers, however, have done just the opposite and have attempted to make an artificial fusion of the subjects. Either extreme is to be condemned.

Another well-known specialist in the teaching of mathematics is Professor W. D. Reeve of Teachers College, Columbia University. He has had considerable experience in three leading universities; namely, the University of Chicago, the University of Minnesota, and Teachers College, Columbia University. He has had first-hand contact with the work in mathematics in each of these institutions. He not only teaches in the graduate school at present but also teaches one high school class so that he can keep in close touch with the high school problems in mathematics. He is author or co-author of textbooks in algebra, general mathematics, and the teaching of junior high school mathematics. At a teachers' meeting a few years ago, he made the following statement:

My proposal is that wherever the conditions permit we arrange our mathematics so that we shall not have courses in algebra, in geometry, or in trigonometry, as such, but a definitely arranged and psychologically ordered

course in mathematics. The plan which we are now formulating will give the student who desires it a four-year course in mathematics in the high school plus one or one and one-half years of college work. It is also hoped that the course will be more compact, will involve less waste, develop more power, and produce even better results than we have been getting from traditional methods. The content of this course will be algebra, plane and solid geometry, trigonometry, most of the analytic geometry, and the fundamental elements of the calculus.

There is nothing essentially new in the plan of teaching algebra and geometry together. The best teaching talent in mathematics the world over, long ago recognized the importance of emphasizing the relation between algebra and geometry and the advisability of teaching them together. The movement to unify the two subjects has been vigorously opposed by those who seriously insist that such unification will break up the logically ordered system of Euclidian geometry and give rise to a series of unrelated ideas without unity or natural sequence. . . .

There is no doubt that the traditional tandem treatment of algebra first, geometry second, and so on, gives rise to a great deal more waste than many teachers realize, and it does not show the intermingling of the subjects as it should. A general mathematics course in the high school will show us more clearly how each subject is reinforced and made clearer and more helpful by the other.[2]

One of the best-known specialists in the teaching of mathematics in our country is Professor J. W. A. Young, of the University of Chicago. On pages 183 and 184 in his recently revised book on the *Teaching of Mathematics*, he made the following statements in regard to his opinion of the relative merits of the two types of mathematics:

The general custom of devoting the first year to elementary algebra exclusively, the second to plane geometry exclusively, the third (or fourth) to solid geometry and the continuation of algebra, and the fourth (or third) to physics, has but little to commend it. Algebra is more abstract than geometry, it has fewer points of contact with real life, its reasoning is more difficult than that of the easier demonstrations of geometry. Further, the more difficult parts of either subject are harder than the easier parts of the other, and each can be made of valuable help in the development of the other. The ideas of both are assimilated slowly by the mind, requiring not so much daily use as occasional use in various ways (quite easy for some time) throughout a long period. To rush a pupil through a subject at high pressure with the exclusion of all other mathematics may train the pupil to pass examinations, but it will not develop the largest measure of thought power or be conducive to real assimilation of the subject.

The conclusion follows naturally that algebra and geometry should be taught side by side; that if under pressure of circumstances either subject must be taken up first, it should be geometry. For many years the tandem order has

[2] "The Case for General Mathematics," *Mathematics Teacher*, 15:381-91 (1922).

been under question, and the weighty objections to it and the strong reasons in favor of another order have repeatedly been urged, though with little general effect. There have been isolated private schools which have ventured on a better order, but they remain exceptional.

The reasons for the failure to make the improvement seem to be those of inertia, of conservatism, rather than a conviction that the change proposed is not good or that the customary order is better. I know of no published defense of the traditional procedure, but still the old order, which finds no defenders in theory, persists in practice.

The writer is impressed by the truth of the last statement in this quotation from Professor J. W. A. Young's book. He has found some criticism of general mathematics, but has never found a genuine defense of the traditional type. Those who oppose general mathematics rarely, if ever, in their writings defend traditional mathematics. Of course the experimental or new type will always have its ardent advocates. It is surprising, however, how much general mathematics in one form or another has been advocated in periodical articles and in books and how little it has been used in the schools.

One criticism, somewhat adverse, of general mathematics is worthy of consideration. It was made by Professor W. W. Hart of the University of Wisconsin. Professor Hart is well known because of the popularity of his high school and algebra textbooks. He expressed his views in regard to general mathematics in the following manner:

To facilitate mastery of the minimum in each grade, unity, emphasis, and coherence must be characteristic of the subject-matter selected for the grade. Certainly, topics or principles which are united somehow are more easily mastered than a lot of disconnected ideas even though they are all mathematical. In short, if there is to be *general mathematics*, common sense dictates that it should be *unified general mathematics*. Mensuration problems of arithmetic, attacked by means of the formula, are a good example of desirable unified mathematics. Another good example is interest problems studied by the formula, or equation, method. On the other hand, it is quite impossible to bring about any vital unification of the concept of congruent triangles and algebra or of constructional geometry and algebra in the ninth grade. . . . When a course becomes so general that it consists of many disconnected topics, then emphasis upon fundamental topics or principles is impossible. One of the virtues of the traditional course in secondary mathematics is that one type of mathematics is emphasized throughout the year, with the result that the pupils who make a slow start often "get on their feet" by mid-year and do satisfactory work by the end of the year.[3]

[3] "Organization of Secondary Mathematics," *School Science and Mathematics*, 23:638-47 (1923).

The last statement in the quotation has little justification. It does not seem likely that a pupil who has made a poor start in a subject like algebra where so much of the second semester's work depends upon what precedes, would make a success of algebra in the latter part of the year. On the other hand, the variety of work in the general mathematics course gives some encouragement to the pupil who is particularly weak in one kind of work. If he is successful in one branch of mathematics, this may help to make him believe that he can master the other branches if he will try hard enough. *Unified* mathematics is good if it will produce enough variety to prevent the pupils from becoming tired of the same or similar kind of work, and if it will give them the best mathematics for the grade in question.

Marie Gugle, Assistant Superintendent of Schools of Columbus, Ohio, was recently president of the National Council of Teachers of Mathematics for two years and is the author of a series of junior high school textbooks in mathematics. She has expressed her views in regard to general mathematics as follows:

> The unified or general mathematics, as reorganized for junior high schools, is far more satisfactory and effective than the old plan of arithmetic in grades seven and eight and algebra in the ninth. Some attempts have been made to work out a general course for the senior high schools, but without success when demonstrative geometry was included. Intuitive geometry for the junior high school fuses beautifully with arithmetic and algebra; but demonstrative geometry is logic and will not fuse. If, however, this subject is offered in the tenth grade as a unit course covering both plane and solid, there is left a wonderful opportunity in the eleventh and twelfth grades for working out a unified course, including algebra, trigonometry, and some analytics and calculus, any part or all of which would more closely articulate with the beginning college mathematics than the present course possibly can, that is, the course in which solid geometry is the last phase of high school mathematics."[4]

The opinion expressed by Miss Gugle in regard to the fusion of demonstrative geometry with other mathematical subjects has been set forth by various other writers. It seems to be a somewhat common notion that demonstrative geometry must be kept distinct from other subjects, even though the others are fused into a general course. Several specialists have expressed themselves in favor of combining plane and solid demonstrative geometry into a year course, especially if intuitive geometry has been taught in the lower grades.

[4] "Revision of College Preparatory Mathematics," *Mathematics Teacher*, 19:321-28 (1926).

One of the schools in which correlated (general) mathematics was a pioneer is the Lincoln High School at Lincoln, Nebraska. In 1902, Edith Long, a teacher in that school, reported that correlated mathematics was already being offered. The work continued for a number of years under her leadership. After she left Lincoln, the work in general mathematics was retarded, but interest was still manifested by some of the teachers. In 1924, Gertrude Jones, a teacher in the Lincoln High School, had the following to say concerning unified mathematics:

> What are some of the advantages of a course in unified mathematics in the junior high school over the traditional courses? In the first place, unified mathematics makes a distinct contribution to each of the three types of pupils it aims to serve. The pupil who leaves school at the end of the ninth grade, or the one who does not continue with the study of mathematics beyond this point has a better mathematical equipment than he would have had by following the traditional course. . . . The pupil who leaves school before completing the ninth grade has the best and most useful mathematical knowledge which he is capable of assimilating and which could be presented to him in the length of time that he has remained in school. Unified mathematics gives to the pupil who will continue his study of mathematics in the senior high school a preview of these courses and a perspective of their content, nature and purpose. For him it prevents an abruptness of transition from elementary school to senior high school mathematics. A course in unified mathematics brings algebra and geometric methods into closer correlation from the start and it furnishes a longer period in which to associate and to practice with these methods. . . .
>
> It is possible and desirable to introduce the more elementary ideas of solid geometry in connection with the related ideas of plane geometry. The work in solid geometry should include numerous exercises in computation based upon the formulas that are established. This will serve to correlate the work with arithmetic and with algebra.[5]

Many other quotations giving the opinions of specialists in the teaching of mathematics could be made. They would be quite similar to the ones already stated. A large majority would show a preference for some kind of general mathematics. The opinions already given are representative of the leading educators who are influencing the thought of our country in regard to the teaching of mathematics.

SUMMARY OF OPINIONS

The writer has examined each article or book listed in the annotated bibliography for opinions in regard to general and tradi-

[5] "Unified Mathematics in Secondary Schools," *Mathematics Teacher*, 17:290-300 (1924).

tional mathematics. No claim is made that this bibliography is complete. It does represent a diligent search through the literature on the subject, and especially that published during the past ten years. A greater effort was made to find opinions favoring traditional than general mathematics. The reason for doing this was that it was easy to find opinions favoring the general type and difficult to find those favoring the traditional type. The bibliography extends back to 1900. A summary of the opinions of the sixty-two writers listed shows that thirty-six favored a more general type of mathematics than was being offered, without specifying any limitations as to the grades; ten favored a more general mathematics course than the one already in the ninth grade, but did not express their opinions in regard to the work of the tenth grade; five favored a more general mathematics course in the ninth grade and traditional mathematics in the tenth grade; four favored traditional mathematics in both grades; and six did not express opinions definite enough to classify. Of the fifty who favored a *more* general type of mathematics in the ninth grade, thirty-four apparently were in favor of making the course completely one of general mathematics. This indicates a great preponderance of published sentiment in favor of a more general mathematics course. It does not show the sentiment of the teachers who are satisfied with things as they are at present. Such teachers as a rule do not discuss the subject. It is doubtful, however, if this type of teachers should be considered in such a study as the present one.

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Describes a course in which the ninth grade mathematics apparently applies the spirit of general mathematics.

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Recommends general mathematics for the junior high school and logically organized courses in algebra, trigonometry, demonstrative, and analytic geometry for the senior high school.

Allen, Gertrude E. "A Reorganized Course of Study for Senior High School Mathematics." *The National Council of Teachers of Mathematics—Second Yearbook*, pp. 256-90 (1927).

Outlines a course in first-year algebra which includes statistical graphs and a unit of numerical trigonometry; also a course in demonstrative geometry which includes an option of five weeks on the mensuration of solids.

Beatley, Ralph. "Effect of Varying the Amounts of Arithmetic, Geometry, and Algebra Studied in the Junior High School." *The National Council of Teachers of Mathematics—Second Yearbook*, pp. 143-53 (1927).

Believes in general mathematics if definition of the term is not too radical.

Breckenridge, W. E. "Applied Mathematics in High Schools. Some Lessons from the War." *Mathematics Teacher*, 12:17-22 (1919).

Advocates a required ninth grade course in practical mathematics which would include the essentials of algebra, geometry, and trigonometry.

Breslich, E. R. "Forward Movements in Secondary Mathematics." *School Review*, 24:282-97 (1916).

Makes a defense of first-year correlated mathematics.

Breslich, E. R. "Junior High School Mathematics." *School Review*, 28:368-78 (1920).

Makes a plea for general mathematics in the junior high school.

Breslich, E. R. "Correlation of Mathematical Subjects." *School Science and Mathematics*, 20:125-34 (1920).

Tells of educators advocating and others opposing fusion of the mathematical subjects. Gives some of the reasons.

Brookman, Thirmuthis. "High School Mathematics." *School Review*, 18:20-28 (1910).

Gives a favorable report of the correlated mathematics in the Lincoln High School at Lincoln, Nebraska.

Cajori, Florian. "Discussion of the Report of the Committee on Algebra." *School Science and Mathematics*, 8:205-8 (1908).

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Clark, J. R. Co-author with J. O. Rugg. See "Rugg," page 158.

Clarke, J. B. "Vital Questions for Teachers of Secondary Mathematics." *School Science and Mathematics*, 6:561-66 (1906).

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Cobb, H. E. "Current Educational Movements and General Mathematics." *School Science and Mathematics*, 16:415-17 (1916).

A discussion favoring general mathematics.

Collier, Myrtie. "The Need of a General Course in Mathematics." *School Science and Mathematics*, 22:845-50 (1922).

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Collins, J. V. "The Perry Idea in the Mathematical Curriculum." *School Science and Mathematics*, 12:296-99 (1912).

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Comstock, C. E. "The Unification of Secondary Mathematics." *Addresses and Proceedings of the National Educational Association for 1904*, pp. 519-21.

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CORNISH, W. A. "The Effort to Make Algebra Yield Fruit." *Mathematics Teacher*, 1:155-59; 2:1-3 (1909).

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DAVIS, ALFRED. "Problems Concerning the Teaching of Secondary Mathematics." *Mathematics Teacher*, 15:467-77 (1922).

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DAVIS, MARGARET ELSIE. "The Teaching of Mathematics in the Junior High School." *Mathematics Teacher*, 13:13-24 (1920).

Favors correlation and unification of the right kind.

DRAKE, L. GILBERT. "Arithmetic in the High School." *Mathematics Teacher*, 15:119-25 (1922).

Advocates a two-year course in general mathematics in the high school, placing particular emphasis upon arithmetic.

DURELL, FLETCHER. "Coöperative Mathematics." *School Science and Mathematics*, 27:905-9 (1927).

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EVANS, GEORGE W. "Some of Euclid's Algebra." *Mathematics Teacher*, 20:128-41 (1927).

States that Euclid's *Elements* was the elements of all known mathematical knowledge rather than geometry alone.

GENTLEMAN, FREDERICK W. "The Purpose and the Content of a Course of Mathematics for Technical and Manual Training High Schools." *Mathematics Teacher*, 6:22-40 (1913).

Outlines a course in general mathematics for the ninth grade, but opposes attempting to fuse algebra and demonstrative geometry.

GUGLE, MARIE. "Mathematics in the Junior and Senior High Schools." *School Science and Mathematics*, 25:29-35 (1925).

Advocates general mathematics for seventh, eighth, and ninth grades. Believes demonstrative geometry must be kept separate. Says there is a wonderful opportunity for a combined course in algebra, trigonometry, and calculus in the senior high school.

GUGLE, MARIE. "Revision of College Preparatory Mathematics." *Mathematics Teacher*, 19:321-28 (1926).

Claims that a general course in mathematics in the senior high school would articulate more closely with beginning college mathematics.

HART, W. W. "Organization of Secondary Mathematics." *School Science and Mathematics*, 23:638-47 (1923).

Expresses some opposition to general mathematics. Says that if general mathematics is offered, it should be unified general mathematics.

HAWKES, H. E. "Educational Values in Mathematical Teaching." *Mathematics Teacher*, 4:75-76, 85-89 (1911).

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HINKLE, E. C. "Algebra in the Junior High School." *School Science and Mathematics*, 25:271-86 (1925).

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JACKMAN, W. S. "Correlation of Mathematics." *Educational Research*, 25:249-64 (1903).

A discussion favorable to correlated mathematics.

JONES, GERTRUDE. "Unified Mathematics in Secondary Schools." *Mathematics Teacher*, 17:290-300 (1924).

A discussion which favors general mathematics and tells of its advantages.

JUDD, C. H. "Combination Mathematics." *Psychology of Secondary Education*, pp. 98-99. Ginn and Company (1927).

Tells why combination (general) mathematics is the better.

LINDQUIST, THEODORE. "Junior High School Mathematics." *Educational Review*, 59:296-303 (1920).

Opposes extreme type of general course but advocates some breaking up of the compartments into which mathematics has been divided.

LONG, EDITH. "Correlation of Algebra, Geometry, and Physics." *Educational Review*, 24:309-11 (1902).

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MCCOY, L. A. "Advantages of a General Course in Mathematics for the First Two Years in the High School." *Mathematics Teacher*, 16: 421-24 (1923).

Advocates general mathematics in the ninth and tenth grades.

MERRILL, H. D. "Fads and Plane Geometry." *Mathematics Teacher*, 20:5-18 (1927).

Says that correlated mathematics has failed to be a success and that demonstrative geometry should maintain its identity and much of its traditional order.

MILLER, G. A. "The Case Method of Teaching Mathematics." *School Science and Mathematics*, 19:344-49 (1919).

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MOORE, E. H. "On the Foundations of Mathematics." *Bulletin of the American Mathematical Society*, 9:402-24 (1903).

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MORRISON, HENRY C. "Reconstructed Mathematics in the High School." *The Thirteenth Yearbook of the National Society for the Study of Education*, Part I, 13:9-31 (1913).

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MYERS, G. W. "Some Respects in Which the Teaching of Elementary and Secondary Mathematics Needs Improvement." *School Review*, 10-87-98 (1902).

A paper setting forth the need of making use of a subject already taught in teaching a new subject and of a wide range of subject matter.

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Contains a good discussion of correlation of subjects.

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A discussion of the first-year and second-year mathematics books developed in the University High School.

MYERS, G. W. "Report on the Unification of Mathematics in the University High School." *School Science and Mathematics*, 11:777-90 (1911).

A discussion of how the mathematical material was unified in the University of Chicago High School.

MYERS, G. W. "Education Movements and General Mathematics." *School Science and Mathematics*, 16:97-105 (1916).

A report is made of the success of correlated mathematics in the University of Chicago High School, and general mathematics is advocated.

MYERS, G. W. "Outstanding Pedagogic Principles Now Functioning in High School Mathematics." *Mathematics Teacher*, 14:57-63 (1921).

Tells of the acceptance of the doctrine of correlation and makes plea for general mathematics.

NEWHALL, CHARLES W. "Correlation of Mathematical Studies in Secondary Schools." *Addresses and Proceedings of the National Educational Association for 1902*, pp. 488-92.

Suggests the advisability of combining algebra and geometry rather than teaching them in parallel courses.

NUNN, T. P. *The Teaching of Algebra (including Trigonometry)*, pp. 19-20, 25–30, 74, 82–91. Longmans, Green and Company, Ltd., London.

Advocates uniting algebra and trigonometry and quite a little intuitive geometry in algebra course.

PERRY, JOHN. *Discussion on the Teaching of Mathematics.* Macmillan and Company, London (1901).

An appeal for practical mathematics rather than the type which is separated into compartments.

PERRY, JOHN. "The Teaching of Mathematics." *Nature*, 62:317-19 (1900).

A syllabus of a course in practical mathematics is given.

PITCHER, ARTHUR D. "The Reorganization of the Mathematical Curriculum in Secondary Schools." *Mathematics Teacher*, 8:1-15 (1915).

Opposes a synoptic (general) course, but favors including trigonometry in the algebra course.

REEVE, W. D. "Unification of Mathematics in the High School." *School and Society*, 4:203-12 (1916).

A plea for unified mathematics in the high school.

REEVE, W. D. "The Case for General Mathematics." *Mathematics Teacher*, 15:381-91 (1922).

Relates some of the advantages of a general mathematics course.

REEVE, W. D. "Significant Changes and Trends in the Teaching of Mathematics in the United States since 1910." *The American Council of Teachers of Mathematics—Fourth Yearbook* (1929).

An excellent exposition of the subject, including some discussion of the general mathematics movement.

RIETZ, H. L. "Is the Present Situation in Regard to the Teaching of Algebra in Our High Schools Satisfactory?" *School Science and Mathematics*, 8:496-502 (1908).

States that the efforts at correlation will make algebra more concrete and broaden its application, even if no decided reorganization of the subject is effected.

RORER, J. T. "Present Tendencies in High School Mathematics." *Mathematics Teachers*, 17:22-30 (1924).

States that examinations by College Entrance Examination Board cause the traditional course to be given, but that the examinations are well within the scope of general mathematics pupils if a year of algebra is given later. Says that the Board has indorsed composite mathematics in a veiled manner.

RUGG, H. O. AND CLARK, J. R. *Scientific Method in the Reconstruction of Ninth Grade Mathematics*. The University of Chicago Press (1918).

Favors general mathematics in the ninth grade at least.

SCHWATT, I. J. "Modern Tendencies in the Teaching of Mathematics." *Mathematics Teacher*, 1:47-77 (1908).

A discussion of the movement to correlate mathematical subjects and an expression of some opposition to it.

SCHORLING, RALEIGH. "General Mathematics." *Mathematics Teacher*, 20:65-81 (1927).

A strong defense of general mathematics.

SHORT, R. L. "The Seventh to Tenth Grades a Unit in Mathematics." *Elementary School Teacher*, 9:471-75 (1909).

Suggests the unification of the work of these four grades instead of the compartment plan of teaching the subjects.

SHORT, R. L. "Discussion on the Unification of Secondary Mathematics." *School Science and Mathematics*, 10:246-48 (1910).

A discussion favoring unified mathematics.

SHOBINGER, J. J. "Practical Suggestions as to Ways of Improving the Mathematical Output of Secondary Schools." *School Science and Mathematics*, 5:401-6 (1905).

Advocates making use of a subject already completed when learning a new subject. States that practical problems almost always require the use of two or three subjects.

SHOUSE, R. D. "Required Mathematics in the Four-Year High School." *School Science and Mathematics*, 22:15-19 (1922).

A plea for general mathematics instead of algebra in the ninth grade.

SKINNER, ERNEST B. "The Arrangement of the Mathematical Studies in the High School Courses—A Comparison with the French System." *School Science and Mathematics*, 10:772-75 (1910).

A claim that the more elementary parts of geometry are better adapted for instruction of children than most of the algebra and that too much algebra is crowded into the first year of high school.

SLAUGHT, H. E. "Aims in Teaching Algebra." *School Science and Mathematics*, 6:105-10 (1906).

States that algebra and arithmetic should be closely connected and that there should be a continual connection of algebra with constructional geometry.

SMITH, D. E. "Movements in Mathematical Teaching." *School Science and Mathematics*, 5:135-39 (1905).

A statement of some of the results of the movement to unify mathematics from the viewpoint of the teachers in the Eastern states.

SMITH, D. E. "Mathematics in the Junior High School." *Educational Review*, 53: 391-96 (1917).

A suggestion of the mathematical content of the junior high school which is essentially general mathematics.

SMITH, D. E. "Introductory Course in Mathematics." *Mathematics Teacher*, 11:105-14 (1919).

General mathematics is suggested for the seventh and eighth grades, followed by courses in algebra, trigonometry, and demonstrative geometry.

SMITH, D. E. "Mathematics in the Senior High School." *The National Council of Teachers of Mathematics—Second Yearbook*, pp. 231-41 (1927).

Suggests survey course in the junior high school, and demonstrative geometry as the chief work of the tenth grade but with at least enough algebra and trigonometry brought in to keep them fresh in mind.

SMITH, EUGENE R. "The Mathematics Curriculum in the Senior High School." *The National Council of Teachers of Mathematics—Second Yearbook*, pp. 242-55 (1927).

Apparently favors general mathematics in the junior high school and traditional mathematics in the senior high school. States that algebra and geometry are two distinct subjects and that it does not seem advisable to try to fuse them.

SNYDER, W. A. "The Middle of the Road." *Mathematics Teacher*, 20:112-22 (1927).

Advocates the traditional courses except for those pupils who will not take mathematics beyond the ninth grade. Says that a course in general mathematics may be better for these.

Stone, Charles A. "Correlation of the Mathematical Subjects Develops Mathematical Power." *Mathematics Teacher*, 16:302-10 (1923).

A report of the results of an investigation of solutions of problems.

Stone, Charles A. "An Approach to the Solution of the Problems which the Traditional Mathematics Program Presents." *Mathematics Teacher*, 18:447-54 (1925).

Advocates an early introduction of algebra and geometry and of beginning the organization of secondary mathematics with the seventh grade.

Story, W. E. "The Unification of Mathematics in School Curriculum." *School Review*, 11:832-55 (1903).

Advocates a greater unity of mathematics in schools.

Swensen, John A. "The Newer Type of Mathematics." *Mathematics Teacher*, 22:152-55 (1929).

A discussion of the New York Regents syllabus in algebra in relation to the recommendation of the National Committee and a plea for the new type (general) mathematics.

Sykes, Mabel. "Radical and Conservative Elements in the Teaching of Mathematics." *School Review*, 13:59-67 (1905).

Advocates some correlation but claims that enough correlation will be obtained if the subject is pushed to its utmost in teaching.

Walsh, C. B. "A Tentative Program of Junior High School Mathematics." *Mathematics Teacher*, 10:85-93 (1917).

Proposes modifications of the mathematics courses to those somewhat more like general mathematics than the traditional courses.

Webb, H. E. "Conditions which Have Led to the Establishment of Junior High Schools." *Mathematics Teacher*, 10:95-9 (1917).

Suggests a generalized course for the seventh, eighth, and ninth grades.

Webb, H. E. "The Future of Secondary Instruction in Geometry." *Mathematics Teacher*, 14:337-41 (1921).

States that if introduction to geometry is given in the junior high school, a combined course in plane and solid geometry might be given in one year. Distinction between plane and solid geometry not essential. Advocates less formalism in geometry.

Webber, P. W. "Combined Mathematics." *Mathematics Teacher*, 14:381-86 (1921).

A defense of general mathematics.

Wright, C. H. "A Discussion of the Report of the Committee on the Unification of Mathematics." *School Science and Mathematics*, 10:242-46 (1910).

A discussion favoring unified mathematics.

Wright, C. H. "Unification of Mathematics in the High School." *School and Society*, 4:561-63 (1916).

Gives a list of textbooks for a unified mathematics course.

Young, J. W. A. *The Teaching of Mathematics in the Higher Schools of Prussia*, pp. 134-35. Longmans, Green and Company (1900).

Makes a statement favoring the parallel teaching of algebra and geometry.

YOUNG, J. W. A. "Concerning Systematic Exposition of Mathematics and the Fusion of its Various Branches in Secondary Instruction." *School Science and Mathematics*, 12:447-56 (1912).

An International Commission report in which the author sets forth the practice of separating mathematics into compartments in America and then recommends simultaneous teaching of the subjects.

YOUNG, J. W. A. *The Teaching of Mathematics*, pp. 97, 183-85. Longmans, Green and Company (1924).

The traditional type of mathematics is condemned and a suggestion made that algebra and geometry be taught side by side.

CHAPTER XI

CONCLUSIONS

OUTSTANDING RESULTS OF THIS STUDY

The results of the present investigation have been given from time to time in the various chapters of the book. There are a few outstanding ones, however, which should be summarized:

1. There is no very clear or definite agreement among mathematicians and general educators as to what constitutes general mathematics. Hence it is necessary to define what is meant by this type of mathematics before it can be discussed intelligently.

2. General mathematics is gradually replacing the traditional type in the seventh, eighth, and ninth grades. No apparent change is being made in the general character of tenth grade mathematics which at present consists almost entirely of plane geometry.

3. General mathematics provides training for college mathematics that is as good as, and perhaps better than, that of mathematics of the traditional type.

4. The indications are that general mathematics creates more interest in the subject than does traditional mathematics.

5. The opinions of high school teachers reveal a large number of reasons for teaching each type of mathematics. General mathematics is favored by many because of the wide variety of information given, because of the interest created, and because of its practical value. Traditional mathematics is favored by many others because of the more thorough knowledge imparted, because of its better organization, and because of the belief that it gives a better preparation for college.

6. Most of those persons who have specialized in methods of teaching mathematics are in favor of a more general type than has been offered.

7. Attempts are being made to correlate the different branches of mathematics. Textbook writers of to-day, however, are careful not to try to fuse material when an unnatural correlation results.

DYNAMIC NATURE OF MATHEMATICS

The organization of mathematics can never become permanently fixed if our school curriculum is to be progressive. The work of the teacher would be greatly simplified if skilled educators could decide experimentally what is the best way to organize the material in any branch of learning. This can rarely, if ever, be done. It is often possible to decide between two particular ways, as this study has attempted to do, but no one can be sure that there are not other ways which have not been considered that are better than either one of those investigated. The wide-awake teacher will always be trying to improve both the organization and content of a subject and the methods of teaching it. The present study has had to do with organization and content. This cannot in any way replace the effect of good methods of teaching. A skillful teacher can secure excellent results with almost any kind of material, but this same teacher can obtain better results with more desirable subject matter. The two go hand in hand, and neither should be considered as an excuse for neglecting the other.

It was impossible for the writer to study the relative merits of traditional and general mathematics as much as he did without forming some definite conclusions and opinions of his own. In the present chapter, he sets forth some of them. They are not offered in the sense of being final. In fact, the writer does not desire them to be considered as more than working bases for further experimentation and study by skillful teachers. He is desirous of keeping mathematics a dynamic subject, for only in this way can it increase its usefulness. Whenever a subject becomes static, it will soon cease to be of very great value in the education of the youth of the country. Our civilization is changing and education must progress as civilization changes.

VALUE OF SUBJECT MATTER INDEPENDENT OF SCHOOL ORGANIZATION

As has already been stated, the American public schools have been organized either on the 8–4 (or 7–4) plan or on the 6–3–3 plan. We have seen that the schools organized according to the 6–3–3 plan have broken away from tradition and have allowed greater freedom in introducing new material into the curriculum. Progressive educators have taken advantage of this freedom and have reorganized the curriculum extensively. The subject matter

in the schools of the 8–4 plan is much more static than in the junior high schools. To the writer, this difference in curricula seems to be unfortunate. If one type of material is better for the pupils in schools of one type of organization, then the same material would surely be better for pupils of the same age and similar environment in schools of the other type. In other words, the organization of the schools does not determine the value of a particular type of subject matter. The sooner American educators can agree that the organization of schools into distinct units does not determine the kind of subject matter to be presented, the better it will be for the youth of our country. The writer is not contending for a definite decision as to the exact subject matter to be offered or its arrangement. He does believe, however, that this false basis of difference in the type of subject matter should be eliminated.

COLLEGE PREPARATION AND WORTH-WHILE MATHEMATICS

Many teachers who insist that traditional mathematics is the better preparation for college advocate general mathematics for those who do not finish high school or for those who do not take mathematics beyond the ninth grade. They admit that general mathematics is more useful to such pupils and will prepare them better to use mathematics in the actual affairs of life. It is unfortunate if there is in reality such a conflict of values between the two types of courses. Surely a kind of mathematics which will function outside of school would also be useful to college students. This conflict of ideas about the relative values of the two courses shows the dominance of the college in our present high school education. Fortunately there *is* no such conflict of values. The prevailing notion in this respect in regard to college preparation is the result of custom and tradition rather than experimental evidence. It is reasonable to demand that, if pupils are to go to college, they should be given the kind of mathematics which will be the best preparation for them. This does not mean, however, that it should be assumed that the present high school and college mathematics is the best that can be offered. The subject matter and arrangement of the courses should take into consideration the entire training of the students, but in such a way that they will receive the best mathematics for any particular year. Whether one leaves school or ultimately completes

a college course, his mathematical education for each year should be made as valuable as possible.

GENERAL MATHEMATICS IN COLLEGE

It has been shown by data in Chapter IV that general mathematics in the first two years of the four-year high school is as good preparation for higher mathematics of the traditional type as is traditional mathematics. The question naturally arises as to what would be the results of a study of the relative merits of the two types as a preparation for *general mathematics* in the eleventh and twelfth grades and in college. It is quite probable that the results would be decidedly different from those obtained in schools in which the higher mathematics was of the traditional type. Until quite recently it would have been difficult to carry on such a study because nearly all of the higher mathematics was of the traditional type. A number of colleges, however, are now introducing general mathematics in the freshman year. Just how extensively this is being done, the writer is unable to say. General mathematics textbooks[1] for the first year of college have been on the market for several years and a new one appears almost every year. In this study, no attempt was made to investigate college mathematics, but the movement is so important in connection with the development of high school mathematics that it deserves mention here. One of the greatest barriers to the growth of general mathematics in the high school has been the traditional type of mathematics in college. If students have been taught traditional mathematics from the earliest grades through the college, it is not to be expected that they will be enthusiastic advocates of general mathematics when they begin to teach. With the introduction of general mathematics in the first year of college, however, more teachers may wish to introduce it in the high school. In this connection, the following quotation from a mathematics teacher in the Iowa State Teachers College is of interest.

[1] J. W. Young and F. M. Morgan, *Elementary Mathematical Analysis.* The Macmillan Company (1917).

R. B. McClenon and W. J. Rusk, *Introduction to the Elementary Functions.* Ginn and Company (1918).

F. L. Griffin, *An Introduction to Mathematical Analysis.* Houghton Mifflin Company (1921).

N. J. Lennes, *Survey Course in Mathematics.* Harper and Brothers (1926).

G. W. Mullins and D. E. Smith, *Freshman Mathematics.* Ginn and Company (1927).

J. E. Rowe, *Introductory Mathematics.* Prentice-Hall, Inc. (1927).

C. H. Currier and E. E. Watson, *General Mathematics.* The Macmillan Company (1928).

In our college, we teach General Mathematics in the freshman year, and are advocating it to our prospective teachers who major in our department. We find that students so trained do better work in our advanced courses and more are taking these courses.

The six state teachers colleges in Oklahoma have revised the freshman curricula for the year 1929 and have provided for a course in general mathematics. This course is to be required of students choosing certain curricula. This revision is typical of the present tendency in the work of the first year of college. No doubt, many other examples could be found similar to the case in Oklahoma.

Most of the textbooks contain the essentials of algebra, trigonometry, and analytic geometry, and an introduction to differential and integral calculus. They are intended for a year's work in mathematics. Most of the authors apparently have tried to unite the material from the different branches to a certain extent but not to break up the unity of the work in attempting to correlate the subject matter. This enables the teacher to present the easier and more important parts of each branch of the subject first.

CORRELATION

The advocates of the correlation of the branches of mathematics claim that it will make mathematics function more readily in the everyday life of the individual. The problems of life do not isolate the different branches, so that one branch will be used in one problem and another in a different problem. In fact, two or three branches of mathematics may be used in the same problem. Since this is the way problem material occurs in life relations, the advocates of correlation insist that it should be taught in school in the same way. On the other hand, those who argue that the branches should be taught separately, say that the teaching of so many branches at the same time is confusing to the pupils and that it does not lead to a mastery of any one of them. Experiments show, however, that practically no greater mastery is gained by spending the entire time on one branch than if it is correlated with other branches. Hence a pupil is not only learning a particular branch of mathematics, but he is also securing considerable knowledge of others at the same time. Psychology teaches that the concomitants are often more valuable than the actual fact or subject that is being taught. In algebra, however,

problems are commonly included which do not advance the knowledge of anything except the algebraic processes involved. Good illustrations of this are the age problems commonly placed in algebra textbooks. The following problem from a current textbook will serve as an example:

> Three years ago Mary was three times as old as John. Three years hence she will be twice as old. Find the present age of each.

Such a problem demands thinking as to how old each was three years ago, how old each will be three years hence, the relation between their ages, and their present ages. All of this is done in order to find two ages which anyone would already know in any real case before he could frame the problem. The work is entirely useless as far as information is concerned. The only thing of value is the training received in algebraic processes and in thinking. If as good training in algebra could be secured by choosing problems in which the attendant facts are worth while, then the value of the learning would be greater. Thus a pupil may learn as much algebra, and at the same time learn considerable geometry or arithmetic or secure other valuable information if the subject matter is properly chosen.

The writer believes thoroughly in correlation. In one sense all problem work involves a certain amount of correlation. The example in regard to the ages of Mary and John involves correlation of algebra with facts concerning the ages of these two persons. The correlation which the writer advocates is the correlation of useful materials or at least those of educational value. Surely the correlation of the different branches of mathematics, such as algebra and intuitive geometry, falls in this category. This does not exclude, however, correlation of such subjects as algebra and physics. The correlation discussed in this study is that between different branches of mathematics rather than between isolated facts or between mathematics and some other school subject, such as physics.

Teachers should be warned against making correlation an end in itself. The fact that some teachers have done this has helped to bring correlation into disfavor with many people. It has often been argued that algebra and demonstrative geometry will not fuse. The attempts to bring about the correlation of subject matter which will not fuse have led to the charge that correlated mathematics produces a forced fusion of the branches of mathe-

matics. Whenever so-called correlations become forced, however, they cease to be correlations but are mixtures which are confusing. In such cases, instead of aiding in learning more mathematics, they break up the unity of mathematics and actually become destructive educationally.

BASES FOR CHOICE OF SUBJECT MATTER

Too often the subject matter has been limited to the particular branch of mathematics which is traditionally offered in the grade in question. Many teachers believe that the seventh and eighth grade pupils should be taught only arithmetic; the ninth grade, algebra; and the tenth grade, plane demonstrative geometry. Such teachers have often no other basis of choice than that it is the subject which is traditionally taught in that particular year.

It is difficult to determine what are the best bases for making the choice of subject matter. There are two which seem to be outstanding in connection with this study. First, the choice of material in each year should be the best possible for pupils if they leave school at the end of that year. To be sure, it often happens that only a small per cent of the pupils quit school without taking another year's work, but the mathematics which they have learned will be just as valuable if they remain in school. Such a choice of subject matter will not confine the work to one branch of mathematics during the year. Second, the easier portions of mathematics should be offered first and the more difficult parts left until later. Many teachers argue with good reason that pupils are not able to complete the more difficult parts of algebra in the freshman year. They claim that this should be left until a later year and the easier portions of geometry and trigonometry taught in its place. This would seem to be good basis for the choice of subject matter. It would certainly decrease either the number of failures in freshman classes or the number who pass with very little knowledge of the subject.

ARRANGEMENT OF SUBJECT MATTER

The arrangement of subject matter should be made psychologically rather than logically. The choice of the best order of learning is far more important than the logical development of the subject as viewed by one who already knows the subject. Pupils cannot see the beauty of an orderly arrangement of material before

they are familiar with the subject matter. Hence most of the effort which teachers make to develop a subject logically in teaching it is wasted time as far as the pupils are concerned.

Particular mention of the tenth grade work should be made in this connection. The purpose of demonstrative geometry is to teach logic. A geometric demonstration is not the best means to convince anyone of a geometric truth, the methods of intuitive geometry being more effective. Hence the methods of proof in a logical demonstration are the chief aim of demonstrative geometry. Many teachers, however, who see the beauty of the whole structure of demonstrative geometry insist that nothing should occur to break the unity of the entire subject. They will not allow anything but geometry to enter into the year's work, and each proposition must build as far as possible upon what has preceded. Any high school pupil who is able to see such unity in geometry while learning the subject is indeed a prodigy. For the most part, each proof is a unit in itself and any break between the propositions does not break into the unity of geometry from the pupil's point of view. The writer mentions this fact to show that if it would seem wise to include some trigonometry or algebra, it could be done without loss to the unity of geometry.

One of the greatest weaknesses of the present arrangement of high school mathematics is that pupils do not know as much algebra at the end of the tenth grade when they are ready to take up Algebra II, as at the end of the ninth grade. It would seem that at least enough algebra should be included in the tenth grade to maintain the ninth grade standard of ability. Psychology teaches that a subject is learned better if the learning is distributed over a long period of time. It is not best to learn a subject, then cease to use it for a considerable length of time, and finally come back to it for concentrated study. It would be much better to distribute the learning over a longer period of time. This is one of the advantages of general mathematics.

INFLUENCES BRINGING ABOUT A CHANGE

There are various influences which may be said to be bringing about a change in the type of mathematics. Among these are the National Committee's Report, the junior high school movement, and the new psychology and philosophy of education. Back of these influences are the specialists in education. Perhaps no

greater cause of progress in education exists than that of the opinion of a few of the leading specialists in the teaching of particular subjects. The best thought of the specialists is nearly always several years in advance of what is being done in the schools throughout the country. Most of the leading educators are expressing opinions favorable to general mathematics. Hence it is reasonable to assume that general mathematics will continue to grow in favor in the schools of America.

PREPARATION OF TEACHERS

Any radical change in the nature of the subject matter and its arrangement should be brought about gradually, unless the teachers are adequately prepared for the change. The failure of general mathematics in many schools has been due to a lack of understanding and appreciation on the part of the teachers. No course can be a complete success without the coöperation of the teachers who are responsible for presenting its contents to the pupils. Most teachers are thoroughly acquainted with the traditional type of mathematics and have been taught in school to believe that it is the proper way of presenting the subject. Hence it is difficult for them to change their convictions in a short time and to believe thoroughly in general mathematics. They will usually be able to make a success of a compromise type, somewhat more general than the kind they have learned. The plan which is being adopted in many colleges, already mentioned in this chapter, of offering general mathematics in the freshman year will have a broadening influence upon young teachers and will help to prepare them to teach general mathematics in the high school.

NEED OF MORE EXPERIMENTATION

Most of the experiments have been confined to investigations which are too limited in scope. Certain types of these will now be discussed and their limitations pointed out. Examples of one type of experiments are those which have been made for the purpose of determining the relative ability in algebra and intuitive geometry of ninth grade classes which have had traditional mathematics and classes which have had general mathematics. Scientific experimentation requires that the pupils in the two groups be equated as to certain kinds of ability. This can be done fairly satisfactory if there are enough pupils from whom to

make the choice. Next, it is necessary for the teachers of the two groups to be equally skillful. Two difficulties are apparent in this respect. First, no valid test of teaching ability has been devised. Second, it would be hard to secure two teachers of equal skill in all respects even if one could tell when such teachers were found. After the classes are ready for the tests, it is not equally fair to the two groups to give them the same algebra and geometry tests. A comparison of the marks made by ninth grade pupils in algebra is not fair to the general mathematics group. An examination in intuitive geometry is equally unfair to the traditional mathematics group. The subject matter which the two groups are taught is different. Therefore no single test can be devised which would test the knowledge of both groups. Such an experiment may prove that pupils who have been taught general mathematics may, for example, learn the traditional material as well as those who have been taught the traditional course, but it will not completely establish the case for general mathematics.

Another type of experiment is one which attempts to determine the relative success of students in college mathematics who have had different kinds of mathematics in the high school. Most of the college mathematics courses are similar to the traditional mathematics in the high school, and hence such a comparison is not equally fair to the groups. If a study were made to determine the relative value of the two types as a preparation for general college mathematics, the results might be quite different. Before any comparison is accepted as conclusive, it will be necessary to determine which is the better type of college mathematics. Then the tests should be given with the better type of college mathematics as the basis for comparison.

A third type of experiment is one for the purpose of determining the relative appreciation of mathematics and the interest created by a study of the subject. If such interest could be accurately measured, the experiment would be a valuable contribution to the teaching of mathematics. The difficulty with this type of experiment has been to eliminate such constant factors as teachers' opinions, college entrance requirements, and the prevalence of traditional courses in higher mathematics. If other things were made equal, the enrollment in elective courses would be a valid basis for determining the interest created by the different types.

Any experiment which attempts to decide the relative merits

of the two types must consider such things as choice of subject matter, efficiency in learning the subject matter, creation of mathematical ability, interest, appreciation, and practical value. A test of any one of these is not enough. A study which includes all of them cannot be completed in a short time. It must necessarily extend over a period of years and take into account the work of junior high school, senior high school, and college. The investigations which are limited in scope are of some value, but they cannot decide the question unless a large number of experiments of various kinds are performed.

SUGGESTIONS FOR TEACHING

On the basis of experiments performed, investigations made, and conclusions which seem reasonable in the light of the facts discovered, the writer offers the following summary of suggestions for the teaching of junior and senior high school mathematics:

1. Teach the best mathematics available for any particular year regardless of what branch or branches it might be.
2. Teach the easier portions of each subject first and leave the more difficult portions for later in the year or for another year.
3. Correlate the branches whenever such correlation seems natural and when one branch can aid in the learning of the other.
4. Teach mathematics in the way it functions in life. Very often the problems in life situations involve two or three of the branches of mathematics. Such problems should be considered in school.
5. Teach the subject matter in a psychological order. Do not wait for each part of the subject to come in the logical development of the mathematics course.
6. Teach mathematics in such a way that the pupils' knowledge of the different subjects will increase each year. It is better to distribute any type of material over a considerable period of time. In this way, the learning will be more effective.
7. Do not let any type of mathematics take the place of efforts to do good teaching.
8. Do not apply one standard for the seventh, eighth, and ninth grades in the junior high school and another for the same grades in schools organized on the 8–4 plan. The organization of the schools does not determine the kind of mathematics which is most valuable to the pupils.

9. Do not have a double standard for pupils who take only one or two years of high school mathematics and for those who expect to go to college. Pupils cannot always decide thus early in the course what their future plans will be. Moreover, there is no valid conflict between the training which is most valuable for the two groups of pupils.

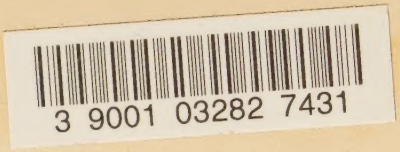
10. Most of the seventh grade work should be in arithmetic and intuitive geometry. Some statistical graphs should be introduced and simple algebraic formulas and equations should be brought in informally.

11. The same subjects should be taught in the eighth grade as in the seventh grade, but the amount of work with algebraic formulas and equations should be considerably increased.

12. The ninth grade work should consist of algebra, numerical trigonometry, and an introduction to demonstrative geometry. Enough demonstrative geometry should be presented to give the pupils an idea of the nature of a *demonstration.* This can probably be done in about six weeks. The major portion of the remainder of the time should be devoted to algebra. Arithmetic and intuitive geometry should not be taught as separate units, but they may be used to strengthen the algebraic work.

13. The core of the tenth grade mathematics should be demonstrative geometry. Perhaps some intuitive solid geometry should be introduced at times in connection with related work in plane geometry. Enough algebra and trigonometry should be included so that the pupils' knowledge of these two subjects will increase rather than diminish, as is so often the case with tenth grade pupils. This can be done for algebra chiefly by correlating it with geometry in the proofs of theorems.

14. A considerable amount of experimentation should be done in connection with the teaching of high school mathematics. Care should be taken that factors do not enter into the experiments which will tend to make the results of no value or which will cause them to be misinterpreted.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